

Name: _____ Section: _____ Score: _____/20

1. We wish to go to a town 21 km away within 15 minutes by car. What is the minimum (average) speed in miles per hour we need? (1 mile = 1.6 km) [5]

$$v \text{ miles/hour} = 21 \text{ km} / 15 \text{ min}$$

This implies

$$\begin{aligned} v &= (21/15)(\text{km/miles})(\text{hour/min}) \\ &= (21/15)(1/1.6)(60/1) = 84/1.6 \\ &= 52.5 \end{aligned}$$

That is, 52.5 miles/hour.

Write simply what you want. We wish to have the speed in miles/hour, and the speed = distance/time.


Handle units just as numbers or symbols

2. Planet A has radius R and mass M_A , and Planet B has radius $2.3R$ and mass M_B . The weight of mass m on the surface of Planet A is W and that on the surface of Planet B is $1.2W$. What is the ratio M_B/M_A ? [5]

The weight of a mass on a planet is the gravitational force acting on the mass when it is on the surface of the planet.

$$W = G m M / R^2$$

Now, let us encode the statements in the question into formulas:

 A: $W = G m M_A / R^2$,

B: $1.2W = G m M_B / (2.3R)^2$.

Let us make the ratio of these two formulas (B over A):

$$1.2W / W = (G m M_B / (2.3R)^2) / (G m M_A / R^2),$$

so

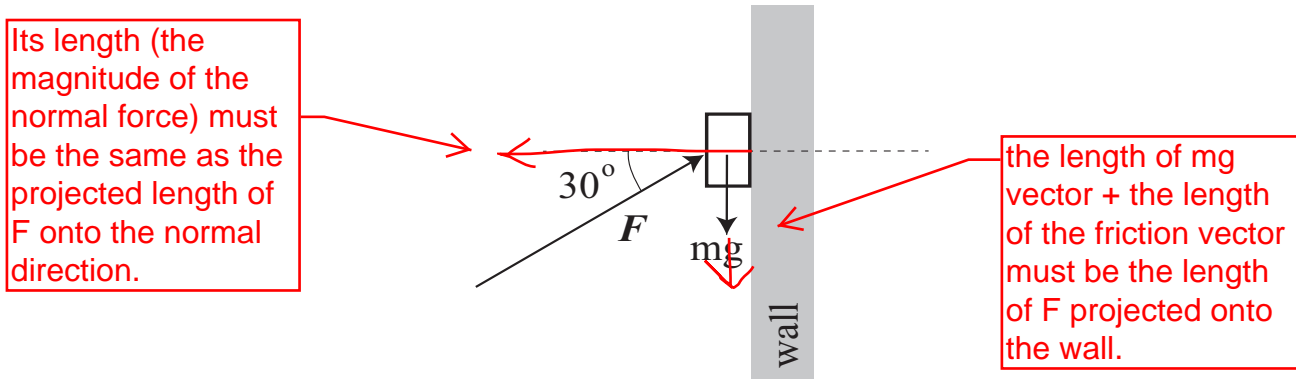
$$1.2 = (M_B / M_A) / 2.3^2.$$

Therefore,

$$M_B / M_A = 1.2 \times 2.3^2 = 6.35.$$

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3. A sanding block of mass m is pushed against a vertical wall with a force of magnitude F with an angle 30° from the normal direction of the wall as illustrated below. The block does not move (due to the static friction).



(a) In the above figure already the force \mathbf{F} and the gravitational force acting on mass m are drawn. Draw the remaining (two) forces acting on the block to complete the free body diagram in the figure above. (Try to be quantitative, using appropriate lengths of arrows. The vectors already drawn are quantitative.) [5]

(b) Find the magnitude of the friction force in terms of $F = |\mathbf{F}|$, m and g . [5]

The total force on the block must be zero. In particular, the sum of the vertical components of the forces must vanish:

$$F \sin 30^\circ = mg + \text{friction force.}$$

Hence,

$$\text{the magnitude of the friction force} = F/2 - mg.$$

Remark: unless the block is on the verge of moving, $f = \mu N$ does not apply. Generally, static friction force is determined by other forces to make the total force vanish.

Name: _____ Section: _____ Score: _____/20

1. Proxima Centauri (the closest star to the sun) is 4.24 light-years away from us. How many years does it take a spaceship with a speed 210,000 miles/h (this speed is of the same order of but faster than the speed of our fastest rocket) to reach this star from the earth? 1 lightyear = 9.5×10^{12} km, and 1 mile = 1.6 km. [5]

Let us honestly write simply what we wish to know:

$$x \text{ years} = 4.24 \text{ ly} / (210,000 \text{ miles/h}).$$

Therefore,

$$\begin{aligned} x &= (4.24/210000)(\text{ly/miles})(\text{h/years}) \\ &= (4.24/210000)(9.5 \times 10^{12} \text{ km/miles})(\text{h}/365 \times 24 \text{ h}) \img alt="yellow speech bubble icon" data-bbox="828 312 858 332" \\ &= (4.24/210000)(9.5 \times 10^{12}/1.6)(1/365 \times 24) \\ &= (4.24 \times 9.5 \times 10^{12}) / (1.6 \times 210000 \times 365 \times 24) = 13685 \end{aligned}$$

That is, about 14 thousand years.

The fastest rocket seems Helios 2 with 157,100 mile/h according to [http://en.wikipedia.org/wiki/Orders_of_magnitude_\(speed\)](http://en.wikipedia.org/wiki/Orders_of_magnitude_(speed))

2. From the following relation

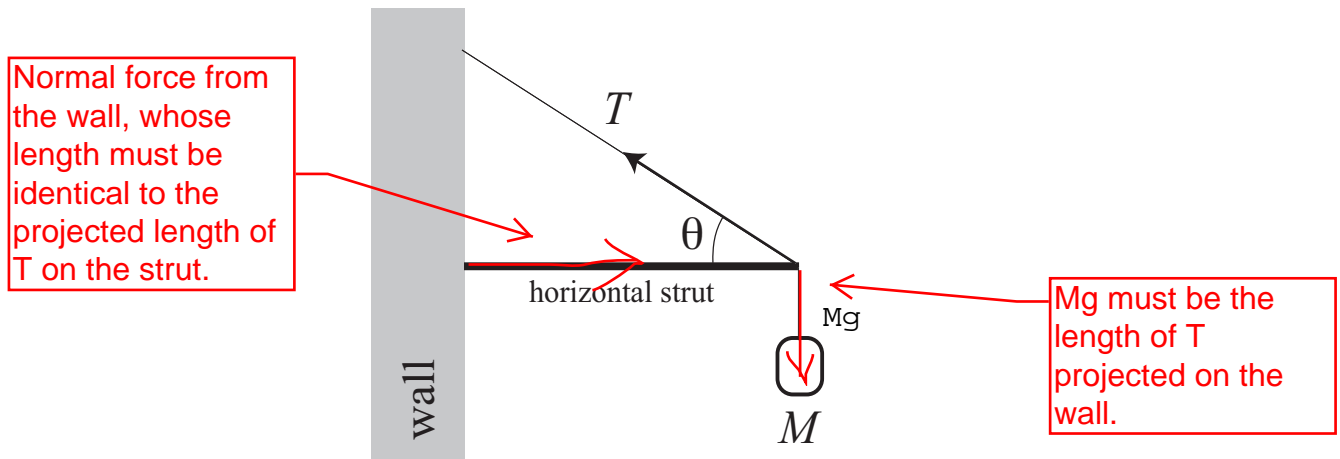
$$\frac{m}{m+M} = 0.25$$

find m/M .

$$4m = m + M, \text{ so } 3m = M \text{ or } m/M = 1/3.$$

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3. A block of mass M is suspended from a horizontal strut with negligible mass. The strut is supported by a cable as illustrated below.



(a) You may assume that the force from the wall on the strut is horizontal. Draw in the figure all the forces (actually, two more forces) acting on the strut. The tension in the string is already drawn.[5]

(b) Find the tension T in the string in terms of M , g and the angle θ . [5]

From the explanations in the above figure
 $T \sin \theta = Mg$, so $T = Mg / \sin \theta$.

Name: _____ Section: _____ Score: _____/20

1. A snail can crawl at a speed of 0.6 cm/s. How many hours does it take the snail to crawl from one goal to the other goal of the football field (100 yards apart)? 1 yard = 0.914 m. [5]

Let us honestly write simply what we wish to know:

$$x \text{ hours} = 100 \text{ yards} / (0.6 \text{ cm/s}).$$

Therefore,

$$\begin{aligned} x &= (100/0.6)(\text{yard/cm})(\text{s/h}) = (100/0.6)(91.4)(1/3600) \\ &= 4.23. \quad 4.2 \text{ hrs.} \end{aligned}$$

(It is 4 hrs and 14 min.)

2. Planet A has radius R_A and mass M , and Planet B has radius R_B and mass $10.5M$. The weight of mass m on the surface of Planet A is W and that on the surface of Planet B is $1.1W$. What is the ratio R_B/R_A ? [5]

The weight of a mass on a planet is the gravitational force acting on the mass when it is on the surface of the planet:

$$W = G mM/R^2$$

Now, let us encode the statements in the question into formulas:

$$\text{A: } W = GmM/R_A^2,$$

$$\text{B: } 1.1W = Gm(10.5M)/R_B^2.$$

Taking the ratio of these two formulas, we obtain (B over A)

$$\begin{aligned} 1.1 &= [Gm(10.5M)/R_B^2]/[GmM/R_A^2], \\ &= 10.5 (R_A/R_B)^2. \end{aligned}$$

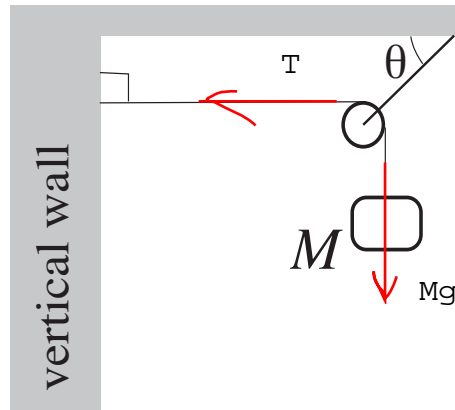
Therefore,

$$R_B/R_A = \sqrt{10.5/1.1} = 3.09.$$



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3. A massless and frictionless pulley hangs from the ceiling by a rope. A block of mass M is suspended by another rope that passes over the pulley and is attached to the vertical wall. The rope end is fixed to the wall with a right angle.

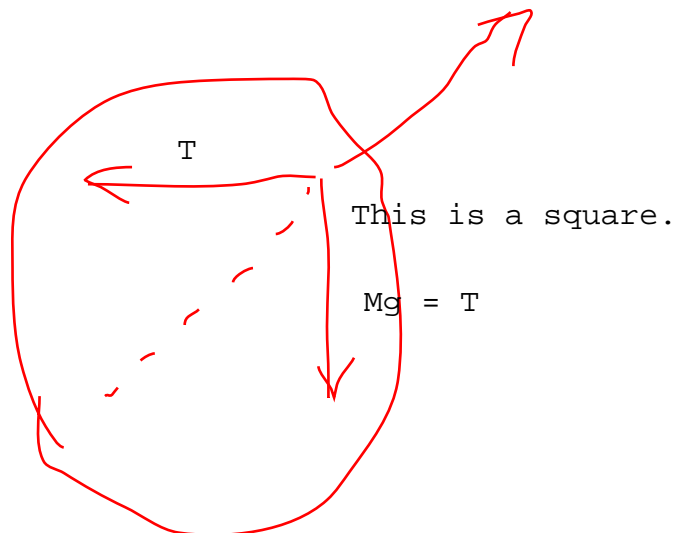


(a) Draw all the forces acting on the massless pulley in the figure. [5]

There is a tension T in the string that must be equal to Mg .

(b) What is the angle θ ? You must justify your answer. [5]

The two forces acting on the pulley are T and Mg . They have the same magnitudes, and they are perpendicular. Then θ must be 45 degrees.



Name: _____ Section: _____ Score: _____/20

1. Suppose we go from Urbana to San Francisco using a hybrid car that can cover 38 km per ℓ . The distance you must drive is 2131 miles. How much does it cost to go from Urbana to San Francisco? Assume that the gas price is \$3.5/gallon. 1 gallon = 3.8 ℓ , and 1 mile = 1.6 km. [5]

Let us honestly write simply what we wish to know:

$$\$x = (\text{volume of needed gas}) \times (\text{gas price/volume})$$

$$= [2131 \text{ miles}/(38 \text{ km}/\ell)] \times (\$3.5/\text{gal}).$$

Therefore,

$$x = (2131/38)(\text{miles}/\text{km})3.5(\ell/\text{gal})$$

$$= (2131/38)(\text{miles}/\text{km})3.5(1/3.8)$$

$$= (2131/38)1.6(3.5/3.8) = 82.6.$$

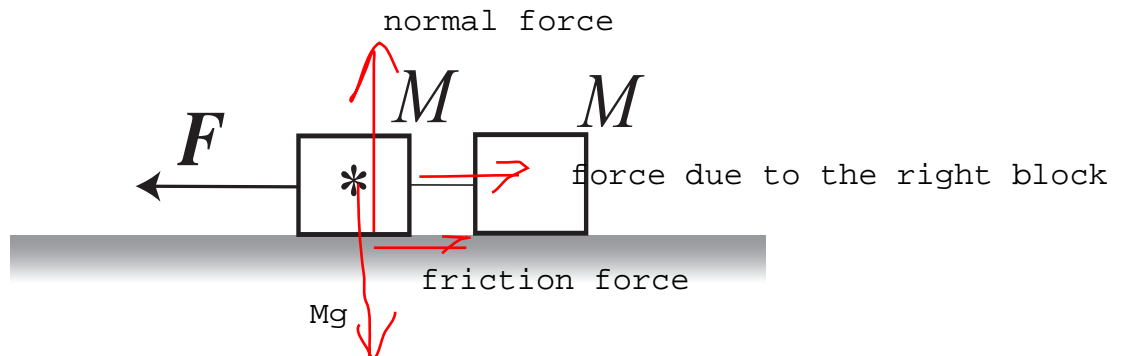
That is, 82.6 dollars.

2. Let $K = (1/2)I\omega^2$, and $L = I\omega$. What is KI/L^2 ? [5]

$$KI/L^2 = (1/2)I^2\omega^2/(I^2\omega^2) = 1/2.$$

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3. Two identical blocks of mass M are on the horizontal floor. They are connected horizontally by a short string, and the left block is pulled by the horizontal force \mathbf{F} to the left. However, due to friction, the blocks do not move. You may assume that the coefficients of static friction between the block and the floor are the same for both the blocks.



(a) Draw all the forces acting on the block with *. [5]

(b) What is the magnitude of the friction force acting on the block with *? [5]

We have only to consider horizontal forces. Let us denote the (magnitude) of the friction force as f , and the force due to the right block as F' . Then, the force balance conditions are:

$$\text{Left block: } F = f + F',$$

$$\text{Right block: } F' = f.$$

Hence, $f = F/2$.