Name: ____________ **Section:** ________ **Score:** ________ **/20**

wave speed   frequency   wavelength

1. WILL Channel 12 TV carrier wave was about 200 MHz. What is the wavelength of this electromagnetic wave in vacuum? The speed of light in vacuum is 3×10^8 m/s. [5]

```
v = f \times \lambda
```

You should memorize this relation.

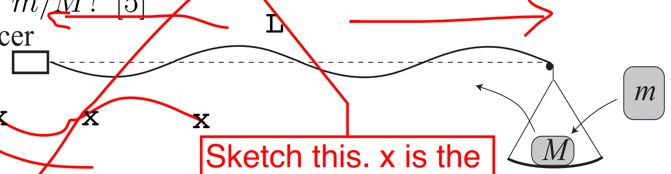
```
\lambda = 3 x 10^8/200 x 10^6 = 300/200 = 1.5 m
```

Try to simplify the numbers before punching them into your calculator.

Do not confuse this M and the M in the formula for v^2 on the formula sheet; the latter M is the mass of the string.

Thus, the tension is Mg.

2. A uniform string is stretched between a transducer of a constant frequency and a smooth peg. The tension in the string is provided by a block of mass M on a massless tray as illustrated below. The number of nodes of the standing wave in the figure is 5 (including both ends). We wish to replace the block of mass M on the tray with another block of mass m (as illustrated) to produce a standing wave with 7 nodes on the string (including both ends). What is the ratio m/M? [5]




```
\lambda
x     x     x     x     x     x     x
```

Sketch this. x is the node

```
Wavelengths
     Five nodes: \lambda_5 = L/2
     Seven nodes: \lambda_7 = L/3.
Since the frequency is the same, v is proportional to the wavelength.
v = \sqrt{F/\mu} [\mu is the linear density of the string], so
   \lambda_7/\lambda_5 = v_7/v_5 = \sqrt{m/M}.
Therefore, (\lambda_7/\lambda_5 = (L/3)/(L/2) = 2/3)
     \sqrt{m/M} = 2/3 or m/M = 4/9.
```

v = f \lambda

In our case F = Mg or mg; F is the tension.

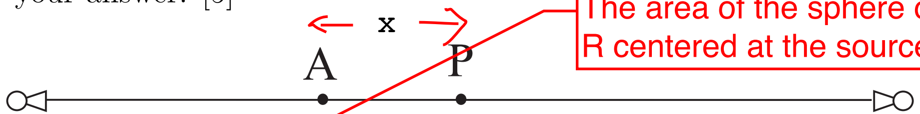
```
Qualitative understanding:
Since f is constant, v and \lambda are proportional, so shorter
wavelength implies slower wave, which implies weaker tension.
```

(**3** and **4** on the next page)

Note for TA:
Truly mathematical justification might seem a bit hard, but if you know convex functions, this is trivial.
Grad students should know the ABC of convex analysis.

3. Let the loudness you hear at the midpoint P of two sirens be β. Now, you move to the point A. Is the loudness you hear at A due to these two sirens larger or smaller than β? You must justify your answer. [5]

The area of the sphere of radius R centered at the source.



The source power

```
We use I = P/(4\pi R^2) in the formula sheet.
In our case, the sirens are the same, so P is a constant you may
ignore.
```

If loudness is large, intensity is large, and vice versa.

```
Instead of comparing the loudnesses, we can compare the intensities
directly.
     At P it is proportional to 1/R^2 + 1/R^2.
     At A it is proportional to 1/(R-x)^2 + 1/(R+x)^2.
Thus, A is always larger.
[How can we prove this? If you draw the graph of 1/R^2 vs R, it is
easy to see 1/(R-x)^2 -1/R^2 >1/R^2 -1/(R+x)^2.]
```

For TA: This is the convexity of 1/R^2 as a function of R.

4. You stand on the roadside and are watching a police car approaching you and then receding from you at a constant speed v along a straight road. When the police car approaches you, you observe its siren frequency as 890 Hz. When the police car is receding from you, the frequency you observe is 790 Hz. What is the speed v of the police car? Assume that the speed of sound is 330 m/s. [5]

observer speed = 0

source speed

Remember the sign convention clearly!

```
This is a Doppler effect problem:
     890 = f(330)/(330 - v),
     790 = f(330)/(330 + v).

The rest is algebra:
This implies
     890 (330 - v) = 330f,
     790 (330 + v) = 330f.
Therefore,
     330(890 - 790) = (890 + 790)v
or
     v = 330 x 100/1680 = 19.64 m/s
```

Here, the speed (magnitude) is written as v. The sign of the velocity must be chosen correctly.

The police car runs in the same direction as the observed sound, so v_{source} = +v.

source downstairs

The police car runs in the opposite direction of the sound being observed, so v_{source} = -v.

Physics 101 (F11) **Q10B**

Name: ____________ **Section:** ________ **Score:** ________/20

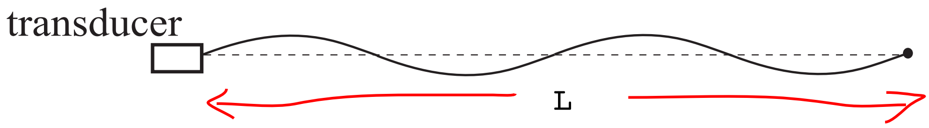
1. The wavelength of a sound wave in medium A is 1.2 m, and that in medium B is 2.3 m. What is the ratio of the sound speeds in these two media, c_A/c_B, where c_A (resp., c_B) is the sound speed in medium A (resp., B)? [5]

You should remember this formula.

```
v = f \times \lambda.
Notice that the frequencies are common. Therefore,
     v proportional to \lambda.
Hence,
     c_A/c_B = \lambda_A/\lambda_B = 1.2/2.3 = 0.52.
```

When a wave of frequency f goes from one medium to another, the frequency is preserved.
Why?
Because `pushing by one medium' causes the wave in the `next' medium;
The wave crest corresponds to the push.

2. A uniform string is stretched with a tension T between a transducer of a constant frequency and a peg. The number of nodes of the standing wave in the figure is 5 (including both ends). We wish to produce a standing wave with 7 nodes on the string (including both ends) by modifying the tension from T to T'. What is the ratio T'/T? [5]



A more detailed explanation may be found in QA.

```
5 nodes: \lambda_5 = L/2
7 nodes: \lambda_7 = L/3.
Since the frequency does not change, the wavelength is proportional
to the wave speed v.
v = \sqrt{T/\mu} [\mu = linear density of the string], so v is
proportional to \sqrt{T}.
Hence,
     wave length is proportional to \sqrt{T}.
Therefore,
     \lambda_7/\lambda_5 = \sqrt{T'/T} = 2/3.
Thus, we get
     T'/T = 4/9.
```

(**3** and **4** on the next page)

source power

The surface are of a sphere of radius R centered at the source

3. The loudness you hear is β from two identical sirens placed 2 m away from you. Now, you prepare 5 of the same sirens and place all of them L m away from you. Then, you observe the identical loudness β as before. What is L? [5]

```
Since \beta = 10 \log_{10}I/I_0, we can directly compare I.

I = P/4\pi R^2 is in the formula sheet.
    old: #=2, R=2, so I \propto 2/2^2 = 1/2.
    new: #=5, R=L, so I \propti 5/L^2
Therefore,
     1/2 = 5/L^2  OR L^2 = 10.
Thus,
     L = \sqrt{10} = 3.16 m
```

Since P is common, I \propto #/R^2

proportional to

source speed

observer speed

4. You are driving a car at a speed 18 m/s along a straight road. Then, a police car catches up and passes you at a constant speed v. You hear the siren frequency change from 650 Hz to 620 Hz. What is the speed of the police car? Assume that the speed of sound is 330 m/s. [5]

Look at the explanation for QA as well

The observer is running in the same direction as the observed sound. v_{obs} = +18

```
Doppler effect

    650 = f (330 – 18)/(330 – v),
    620 = f (330 + 18)/(330 + v)

The rest is mere algebra:
From this, solving them for f, we get
    (650/312)(330 - v) = (620/348)(330 + v)
or
    2.083(330 - v) = 1.7816(330 + v).
That is,
    0.301x330 = 99.3 = 3.865v  so v = 25.7 m/s.
```

The observer is running against the observed sound, so v_{obs} = -18

The source (police car) is running in the same direction as the observed sound. v_{source} = + v

The source is running in the opposite direction of the observed sound wave, so v_{source} = -v.

Name: ____________ **Section:** ________ **Score:** ________/20

1. A gravitational wave produced by a supernova propagates at the speed of light ($= 3 \times 10^8$ m/s). The frequency is 1.2 GHz. What is the wavelength? [5]

See QA.

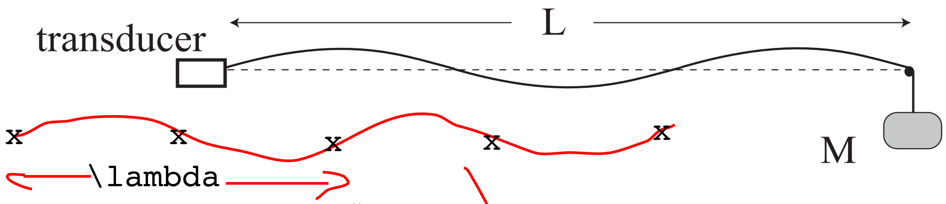
```
v = f \lambda
3 x 10*8/1.2 x 10^9 = 3 x 10*8/12 x 10^8 = 1/4 = 0.25 m.
```

Do not confuse this M and the M in the formula for v^2 on the formula sheet; the latter M is the mass of the string.

f is constant.

Thus, T = Mg, but in this problem this is fixed.

2. A uniform string of linear density μ_0 is stretched with a tension T between a transducer of a constant frequency and a peg, and the tension is provided by a mass M. The number of nodes in the figure is 4 (including both ends). We wish to to produce a standing wave with 5 nodes on the string (including both ends) by replacing the string with a different linear density μ_1. What is the ratio μ_1/μ_0? [5]



Sketch this

```
Key point: changing the # of nodes implies changing the
wavelength, which implies changing the wave speed (if the
frequency is constant).  So, let us study the wavelength:
     4 nodes: \lambda_4 = 2L/3
     5 nodes: \lambda_5 = L/2
Hence, (from \lambda \propto v)
     \lambda_5/\lambda_4 = v_5/v_4 = (L/2)/(2L/3) = 3/4.
We know v = \sqrt{T/\mu}.  Here, T is kept constant, so
     v_5/v_4 = \sqrt{\mu0/\mu1} = 3/4.
Therefore,
     \mu1/\mu0 = 16/9.
```

\propto implies `proportional to'

(**3** and **4** on the next page)

```
Qualitative understanding:
f is the same, so shorter wavelength corresponds to slower wave speed.
If the tension is the same, `heavier wire' gives slower propagation
speed.
Therefore, to shorten the wavelength, the wire should be heavier under
the constant frequency and tension.
```

See the explanation in QA and B

3. The loudness you hear is β from three identical sirens placed 3 m away from you. Now, you prepare 5 of the same sirens and place all of them 5 m away from you. The loudness you hear now is β'. What is the difference $\beta' - \beta$? [5]

```
# = 3, R = 3
     I = 3 (P/4\pi 3^2) = (1/3)(P/4\pi).
# = 5, R = 5
     I' = 5(P/4pi 5^2) = (1/5)(P/4\pi).
Therefore,
     \b = 10 log_{10}(I/I_0),
     \b' = 10 log_{10}(I'/I_0),
so
     \b' - \b = 10 log_{10}(I'/I)
              = 10 log_{10}(3/5) = -2.2 (dB).
```

This is before the encounter

the observer speed

source speed

4. You are driving a car at a speed of 18 m/s along a straight road. Then, a police car comes along the opposite lane toward you and passes you. You hear the siren frequency change from 850 Hz to 620 Hz. What is the speed v of the police car? Assume that the speed of sound is 330 m/s. [5]

Read an explanation in QA

The source (police car) is running in the same direction of the observed sound propagation, so v_{source} = +v.

The observer is running against the sound observed, so v_{obs} = -18

```
Doppler effect
     850 = f (330 + 18)/(330 - v) = 348f/(330 - v),
     620 = f (330 - 18)/(330 + v) = 312f/(330 + v).
```

f is solved from both the equations

```
The rest is mere algebra:
From this
     (850/348)(330 − v) = (620/312)(330 + v).
or
     2.4425(330 − v) = 1.9871(330 + v),
     0.4554 x 330 = 4.885v
Hence,
     v = 30.76 m/s (110 km/h).
```

Name: ______________ **Section:** __________ **Score:** __________/**20**

1. What is the frequency of light of wavelength $\lambda = 350$ nm? The speed of light is 3×10^8 m/s and 1 nm $= 10^{-9}$ m. [5]

See QA.

```
v = f \lambda
f = 3 x 10^8/350 x 10^{-9} = 8.57 x 10^{14} Hz.
```

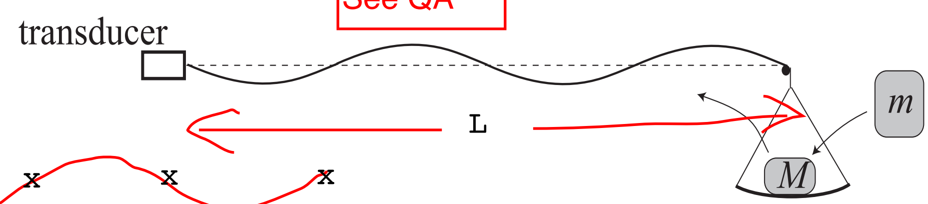
Do not confuse this M and the M in the formula of v^2 in the formula sheet.

f is constant here.

Thus, the tension is given by Mg or mg in this problem.

2. A uniform string of linear density μ_0 is stretched with a tension T between a transducer of a constant frequency and a peg, and the tension is provided by a mass M. The number of nodes in the figure is 5 (including both ends). We wish to produce a standing wave with 4 nodes on the string (including both ends) by replacing the mass M with another mass m. What is the ratio m/M?[5]

See QA



Sketch this.

```
x         x        x         x
\lambda
Wave speed is v = \sqrt{Mg/\mu} [\mu = linear density of the string].
We keep the frequency, so the wave length is proportional to the
wavelength.
   Let us study the wave length:
     \lambda_5 = L/2,
     \lambda_4 = 2L/3.
So,
     \lambda_5/\lambda_4 = v_5/v_4 = (L/2)/(2L/3) = 3/4
On the other hand, v \propto \sqrt{M}, so
     v_4/v_5 = \sqrt{mg/Mg}.
That is,
     m/M = 16/9.
```

This is the ratio of the tensions.

(**3** and **4** on the next page)

```
Qualitative understanding:
Since the frequency does not change, longer wavelength implies
faster wave speed.  If the string is the same, stronger tension
implies faster wave speed.  Therefore, to make wavelength longer
we need a heavier weight.
```

You can directly compare intensities.

In the formula sheet we find I = P/4\pi R^2. See QA for an explanation. In our problem P is the same for all the sirens. Therefore, total I is proportional to # / R^2.

3. Which is larger, the loudness β_1 due to 100 identical sirens placed at 50 m or that β_2 due to 200 sirens (the same ones as before) placed at 70 m? [5]

```
# = 100, R = 50 m
     I = 100(P/4\pi 50^2) = (1/25)(P/4\pi) = (2/50)(P/4\pi) .
# = 200, R = 70 m
     I' = 200(P/4\pi 70^2) = (2/49)(P/4\pi).
Instead of comparing \beta we can compare I, since log is a monotone
increasing function.
     I < I',
so
     \beta_2 > \beta_1.
```

It should have been clearly stated, but I guess you understand that the observer is standing still. v_{obs} = 0.

The source speed.

4. On a salt flat a car propelled by a jet engine reaches a speed of $c/3$, where c is the speed of sound. The car zips past you along a straight trajectory. A siren is placed on the car. What is the ratio of the frequency f_A you hear while the car is approaching and the frequency f_R you hear while the car is receding from you? [5]

See QA for further explanation/memo.

```
Doppler effect
     f_A = f c/(c - c/3) = 3f/2,
     f_R = f c/(c + c/3) = 3f/4.
Hence,
     f_A/f_R = 2.
```

The source is running with the sound in the same direction, so v_{source} = +c/3

Now the sound you hear and the car are running in the opposite directions, so v_{source} = -c/3.