

Name: _____ Section: _____ Score: _____/20

1. Let c be the speed of light in the air. Its wavelength is 680 nm in the air. It goes into a liquid in which the speed of light is $0.7c$. What is the wavelength of the light in this liquid? [5]

$$f \lambda = v = \text{wave speed}$$

f is common to all the media, so λ is proportional to the wave speed.

$$c/680 = 0.7c/\lambda, \text{ so } \lambda = 0.7 \times 680 = 476 \text{ nm.}$$

This is a proportionality relation, so as long as you use the same units, any unit will do, m, nm, cm, inch, miles, etc.

2. Between a transducer and a peg is a string as illustrated below. With the frequency f_0 of the transducer, there are 5 nodes on the string (including both the ends).



We wish to reduce the number of nodes to 4 (including both the ends) by retuning the transducer frequency to f . What is f/f_0 ? Assume that the tension in the string is kept constant. [5]

$$5 \text{ nodes: } \lambda = L/2, f_0 L/2 = v,$$

$$4 \text{ nodes: } \lambda = 2L/3, f(2L/3) = v.$$

$$\text{Therefore, } 2f/3 = f_0/2, \text{ or } f/f_0 = 3/4.$$

If you feel uncertain, sketch the wave.

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¹edited by Alex Weiss

3. A siren produces a sound of loudness β_5 5 m away from it. Now, two of the identical sirens as this are placed 10 m away from you. What is the new loudness β you hear in terms of β_5 ? [5]

$$\beta_5 = 10 \log[I(1)/5^2 I_0]$$

$$\beta = 10 \log [2I(1)/10^2 I_0]$$

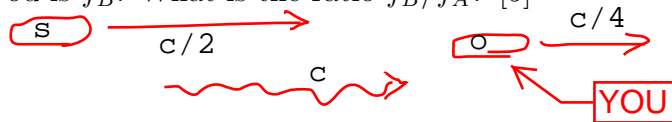
IN the formula sheet you find $I = P/4\pi R^2$. This implies $I(R) = I(1)/R^2$, where $I(R)$ the intensity R away from the source.

Therefore,

$$\beta - \beta_5 = 10 \log[2 \times 5^2 / 10^2] = 10 \log (5/10)$$

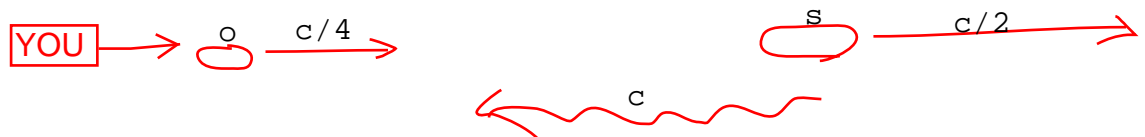
$$= -10 \log 2 = -3 \text{ dB.}$$

4. When you are flying a plane along a straight line at a speed $c/4$, a jet plane catches up to you and then passes you at a constant speed of $c/2$. In this problem, c denotes the speed of sound. The sound of the jet engine you hear before it catches up to you is f_A and after it catches up to you is f_B . What is the ratio f_B/f_A ? [5]



Before: as you see from the above figure, o and s are moving WITH the sound of interest, so $v_o = +c/4$, $v_s = +c/2$.

$$f_A = f_0 (c - c/4)/(c - c/2) = f_0(3/4)/(1/2) = 3f_0/2$$



After: as you see from the above sketch, o and s are now moving AGAINST the sound of interest, so $v_o = -c/4$, $v_s = -c/2$.

$$f_B = f_0 (c + c/4)/(c + c/2) = f_0(5/4)/(3/2) = 5f_0/6.$$

Therefore, $f_B/f_A = (5/6)/(3/2) = 10/18 = 5/9$.

Name: _____ Section: _____ Score: _____/20

1. Let λ_0 be the wavelength of a sound in a medium in which the speed of sound is 320 m/s. This same sound goes into another medium in which the speed of sound is 980 m/s. Its wavelength is now λ . What is the ratio λ/λ_0 ? [5]

$f\lambda = v = \text{wave speed.}$

The frequency is independent of media, so v/λ is constant, or v is proportional to λ .

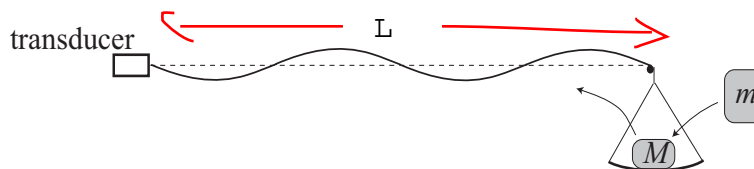
$$320/\lambda_0 = 980/\lambda$$

or

$$\lambda/\lambda_0 = 98/32 = 3.06.$$

This is a proportionality relation, so as long as you use the same unit (say, nm or inch), any unit will do.

2. A uniform string is stretched between a transducer of a constant frequency and a smooth peg. The tension in the string is provided by a block of mass M on a massless tray as illustrated below. The number of nodes in the figure is 5 (including both ends).



We wish to replace the block of mass M on the tray with another block of mass m (as illustrated) to produce a standing wave with 3 nodes on the string (including both ends). What is the ratio m/M ? [5]

5 nodes: $\lambda = L/2$.

3 nodes: $\lambda = L$. Sketch the modes.

In this case, f is kept constant, so

$$\lambda \propto v = \sqrt{Mg/\mu}$$

That is, λ/\sqrt{M} is constant:

$$(L/2)/\sqrt{M} = L/\sqrt{m} \text{ so } \sqrt{m/M} = 2$$

This implies $m/M = 4$.

Or, as long as the frequency is the same, the wave length is proportional to the wave speed. On the other hand, the wave speed is proportional to \sqrt{M} , so the wavelength $\propto \sqrt{M}$.

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¹edited by Alex Weiss

3. A siren produces a sound of loudness β_{10} when placed 10 m away. Now, four sirens identical to this are placed 20 m away. What is the new loudness β in terms of β_{10} ? [5]

$$\beta_{10} = 10 \log(I_1/10^{-12})$$

See A for a detail.

$$\beta = 10 \log(4I_1/20^{-12})$$

Do not confuse β and I . Only I is additive.

$$\begin{aligned} \beta_{10} - \beta &= 10 \log(20^{12}/4 \times 10^{12}) \\ &= 10 \log 1 = 0. \end{aligned}$$

Be sure that you are comfortable with logarithms. In these days basic biological observables are often log of the raw data as in the microarray data. Also chemists should know the Labert's law.

4. A siren is on the roadside of a straight road, on which you drive a car at a constant speed v . When you are approaching it, the frequency you hear is 710 Hz. When you are receding from the siren, the frequency you hear is 640 Hz. What is your speed v ? Assume that the speed of sound is 330 m/s. [5]

You are against the sound you hear.

Before: $v_o = -v$; $v_s = 0$, so $710 = f_0(330 + v)/330$,
 After: $v_o = +v$; $v_s = 0$, so $640 = f_0(330 - v)/330$.

Therefore,
 $71/64 = (330 + v)/(330 - v)$.

Now, the sound is from behind, so you are running WITH the sound you hear.

Or,
 $1.109(330 - v) = 330 + v$, that is
 $366 - 1.109v = 330 + v$.

Therefore,
 $36 = 2.109v$, that is, $v = 17.07$ m/s (ca 38 mph).

Name: _____ Section: _____ Score: _____/20

1. The wavelength of a sound wave is 30 cm in a medium in which the speed of sound is c_0 . This same sound wave propagates into another medium in which the speed of sound is c . Its wavelength is now 140 cm. What is the ratio c/c_0 ? [5]

$f \lambda = \text{wave speed}$. The frequency does not depend on media, so $v/\lambda = \text{constant}$. That is,

$$c_0/30 = c/140,$$

so

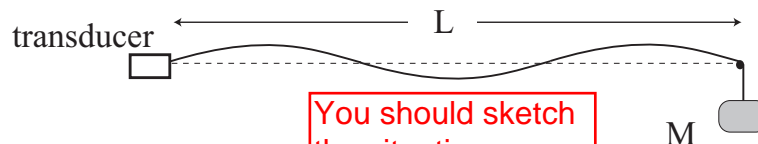
$$c = (14/3)c_0 \rightarrow c/c_0 = 14/3.$$

Notice that the wave speed c is proportional to λ .

As long as you use the same unit, any unit is OK, because it is a proportionality relation

Even if c appears, don't always interpret it as the speed of light.

2. A uniform string of linear mass density μ_0 is stretched between a transducer of a constant frequency and a smooth peg. The tension in the string is provided by a block of mass M . The number of nodes in the figure is 4 (including both ends).



You should sketch the situation,

Keeping the mass M and the transducer frequency, we wish to have 5 nodes on the string (including both ends) by replacing the string with a new one of a linear mass density μ . What is the ratio μ/μ_0 ? [5]

The frequency is preserved, so $f = v/\lambda$. If the linear mass density is altered, the sound speed c along the string is altered. $c = \sqrt{F/\mu}$, but $F (= Mg)$ is fixed, so c is proportional to $1/\sqrt{\mu}$.

4 nodes $\rightarrow \lambda = 2L/3$; 5 nodes $\rightarrow \lambda' = L/2$

$$c_0/(2L/3) = c/(L/2), \text{ so } c/c_0 = 3/4$$

This implies

$$c/c_0 = \sqrt{\mu_0/\mu}.$$

Of course, we could simply recall $c \propto \lambda$

Therefore, $\mu/\mu_0 = 16/9$.

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¹edited by Alex Weiss

3. A cannon makes a sound of loudness β at a distance 50 m away from the point where it is fired. Now, four cannons, identical to the first, are placed a distance L away from you and are fired simultaneously. You hear the same loudness β . What is the distance L ? [5]

$$\beta = 10 \log(I_1/50^2 I_0).$$

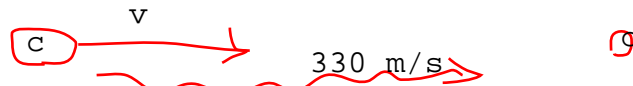
$$\beta = 10 \log(4I_1/L^2 I_0).$$

(1) $I(r) = I(1)/r^2 = P/4\pi r^2$.
 (2) For independent sources, I_i are additive. E.g., from 3 cannons implies $3I_{\text{single}}$

You may directly compare the intensities.

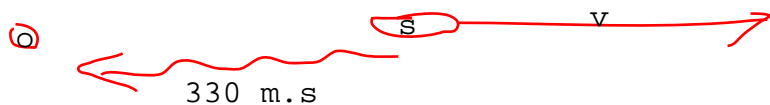
Clearly, These two equalities imply $50^2 = L^2/4$, so $L = 100\text{ m}$.

4. You stand on a roadside and are watching a police car at a constant speed v approaching you and then receding from you along a straight road. When the police car approaches you, you observe its siren frequency as 710 Hz . When the police car is receding from you, the frequency you observe is 640 Hz . What is the speed v of the police car? Assume that the speed of sound is 330 m/s . [5]



Before: $v_o = 0$; The source of the sound you hear is moving WITH the sound, so $v_s = +v$.

$$710 = f_0 \cdot 330 / (330 - v);$$



After: $v_o = 0$; The source of the sound you hear is moving in the opposite direction to that of the source itself, so $v_s = -v$.

$$640 = f_0 \cdot 330 / (330 + v).$$

Therefore,

$$71/64 = (330 + v)/(330 - v).$$

The same as B4

There might be a cleverer way, but let's use brute force:

$$(71/64) (330 - v) = 330 + v, \text{ that is, } 365.97 - 1.109v = 330 + v, \text{ so}$$

$$36 = 2.109v \rightarrow v = 17.07\text{ m/s}$$

Name: _____ Section: _____ Score: _____/20

1. Let c be the speed of light in the air. Its wavelength is 680 nm in the air. It goes into a liquid in which its wavelength is 510 nm. What is the speed of the light in this liquid in terms of c ? [5]



$$f\lambda = v \text{ (wave speed).}$$

Since f is the same for any media, v/λ is independent of the medium. Therefore,

$$c/680 = v/510,$$

so $v = c(51/68) = c(3/4)$ that is, $3c/4$.

This is a proportionality, so any unit may be used.

2. Between a transducer and a peg is a string as illustrated below. With the frequency f of the transducer, there are 5 nodes on the string (including both the ends).



Now, the frequency of the transducer is increased to $1.5f$. How many nodes are there on the string (including both ends)? [5]

$$L = 2 \lambda$$

$$v = f \lambda$$

so

$$f(L/2) = 1.5f \lambda$$

implies

$$\lambda = L/3,$$

There must be 7 nodes. There are 3 wavelengths on L . Sketch the figure!

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¹edited by Alex Weiss

3. A siren produces a sound of loudness β at a distance 5 m away from it. Now, two sirens identical to this one are placed a distance L away from you and you hear the same loudness β . What is the distance L ? [5]

everything after / is downstairs

$$I(R) = P/4\pi R^2$$

$$\beta = 10 \log(I_5/I_0) = 10 \log(I_1/5^2 I_0)$$

The new situation is

$$\beta = 10 \log(2I_1/L^2 I_0)$$

Two ingredients are used:

(1) I at distance r is $I(r) = I(1)/r^2$

(2) From independent sources I_i are additive: $I = \sum I_i$

Therefore,

$$10 \log(I_1/5^2 I_0) = 10 \log(2I_1/L^2 I_0)$$

That is,

$$\log(1/5^2) = \log(2/L^2).$$

Therefore,

$$L^2 = 2 \times 5^2 \rightarrow L = \sqrt{2} \times 5 = 7.07 \text{ m.}$$

We can discuss I instead of β , so you can immediately get

$$I_1/5^2 I_0 = 2I_1/L^2 I_0.$$

4. On a salt flat you watch a speed test of a car with a jet engine. The car reaches a speed of $c/2$, where c is the speed of sound. On the car is a siren and you can hear the frequency f_A when the car is approaching you and that f_R when the car is receding from you. What is the ratio of f_A/f_R ? [5]

Before: $v_o = 0$; $v_s = +c/2$, so $f_A = f_0 \frac{c}{(c-c/2)} = 2 f_0$.

After: $v_o = 0$; $v_s = -c/2$, so $f_R = f_0 \frac{c}{(c+c/2)} = 2f_0/3$.

Therefore, $f_A/f_R = 2 f_0 / (2f_0/3) = 3$.