Physics 101 (S11) **Q11A**[1]

Notice that
(1 + \beta \Delta T)
= (1 + \alpha \Delta T)^3
= (1 + 3 \alpha \Delta T)

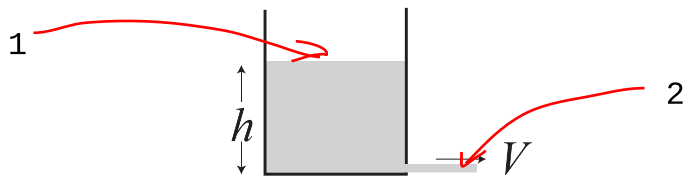
Name: ______________ **Section:** __________ **Score:** __________/**20**

1. Let $\rho(T)$ be the density of a material at temperature T. Its linear thermal expansion coefficient is $\alpha = 1.7 \times 10^{-5}$ K^{-1}. Find $1 - \rho(450K)/\rho(300K)$. [5]

```
Suppose the volume thermal expansion coef is \beta.  Then, the unit volume
at temperature T becomes 1 + \beta \Delta T at temperature T + \Delta T.
Therefore, the density
     \rho(T + \Delta T) = \rho(T)/[1 + \beta \Delta T].
We know \beta = 3 \alpha.  Thus,
     1 - \rho(T)/\rho(T_0) = 1 - 1/[1 + 3\alpha(T - T_0)],
                           = 3\alpha(T - T_0)
                           = 3 x 1.7 x 10^{-5} x 150
                           = 7.65 x 10^{-3}
```

1/(1 + x) is almost
1 - x. Recall
(1 + x)^a = 1 + a x

2. At the bottom of a container is a small hole. The height of the liquid surface is h in the container as illustrated. Write down the speed V of the liquid coming out of the bottom hole in terms of the liquid density ρ, g (acceleration of gravity) and h. [5]




```
This is a very standard Bernoulli: P + (1/2)\rho v^2 + \rho gh = const.
1: P_A + 0 + \rho g h
2: P_A + (1/2)\rho V^2 + 0
These two must be identical, so
     (1/2)V^2 = gh, or V = \sqrt{2gh}.
```

P_A = atmospheric
pressure

(**3** on the next page)

[1]edited by Alex Weiss

3.

a. In a container of volume 0.2 m^3 is 4.6 kg of an ideal gas at Pressure 1.5×10^6 Pa and temperature $T = 300$ K. What is the molecular mass of the gas? [5]

```
Let us first compute the quantity of the gas in moles n in the container:
     n = PV/RT.
If you use R = 8.31, P must be in Pa, V in m^3 and T in K. Therefore,
     n = (1.5 x 10^6) x 0.2/8.31 x 300 = 120.3 moles.
The molecular mass is the mass of 1 mole in grams, so
     4600/120.3 = 38.2 amu
```

BE CAREFUL.

b. We wish to reduce the root mean square velocity of the molecules in the container by 5%. What is the new temperature? [5]

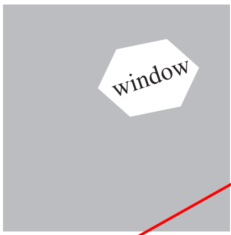
```
v_{rms} is proportional to \sqrt{T}.  Therefore,
v_{rms} ->  0.95 v_{rmp}, so T -> 0.95^2 T
That is, 0.95^2 x 300 = 270.75 K.
```

The total kinetic energy K is proportional to T. K = (1/2) m v_{rms}^2, so
v_{rms} is proportional to \sqrt{T}.

Physics 101 (S11) **Q11B**[1]

Name: ________________ **Section:** ________ **Score:** ________/**20**

1. In a large square made of a uniform substance is a window of area 5.3 m^2 as illustrated below at $T = 350$ K. with (linear) thermal expansion coefficient $\alpha = 1.7 \times 10^{-5}$ K^{-1}.

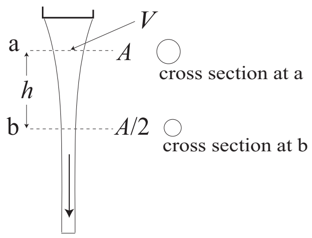


This 2 is the dimension of the object.
Recall (1 + x)^a = 1 + a x very accurately.

What is the change of the area of the window, if $T = 500$ K? [5]

```
We know that the window expand just as the `wall' part, so the area A(T)
obeys
     A(T) = [1 + 2 \alpha (T - T_0)]A(T_0)
That is, the change is
     2 \alpha (T - T_0)A(T_0) = 2 x 1.7 x 10^{-6} x 150 x 5.3
                              = 0.027 m^2
```

2. From a faucet comes out a circular column stream of water as sketched below. At the location a (see sketch), the speed of water is V and the cross section of the water column is A. At a distance of h below below point a is a point b. At location b, the cross section of the water column is $A/2$.



P_A = atmospheric pressure

Obtain V in terms of h, ρ (the density of water), g. [5]

```
Mr Bernoulli can solve this: P + (1/2)\rho v^2 + \rho g h = const.
a: P_A + (1/2)\rho V^2 + \rho g h
b: P_A + (1/2)\rho V_b^2 + 0.
These two expressions must be the same. We need V_b. Continuity
equation implies AV = (A/2)V_b, so V_b = 2V.  Therefore,
     (1/2)\rho V^2 + \rho g h = (1/2)\rho (2V)^2
Thus,
     gh = (3/2)V^2 -> V = \sqrt{2gh/3}.
```

(**3** on the next page)

[1]edited by Alex Weiss

3.

a. In a container of volume 0.2 m^3 is 129 moles of an ideal gas at temperature $T = 300$ K. What is the pressure P of the gas? [5]

If R = 8.31 is used and V = 0.2 m^3, T in K, then P is in Pa.

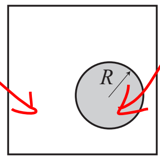
```
P = nRT/V = 129 x 8.31 x 300/0.2
     = 1,607,985 Pa.
It is about 1.6 x 10^6 Pa.
```

b. We wish to reduce the root mean square velocity of the molecules by 5% without changing the volume of the container. Let P' be the pressure required to make change. What is the ratio P'/P, where P is the pressure in **a** of the gas. [5]

```
To change the rms velocity, we must change the temperature T.
T  = PV/nR, but n and V are kept constant, so to change T we must
change P.  Since P is proportional to T, and since v_{rms} is
proportional to \sqrt{T}, P is proportional to v_{rms}^2.
     Therefore,
          P'/P = 0.95^2 = 0.90.
```

Name: ____________ **Section:** ________ **Score:** ________/**20**

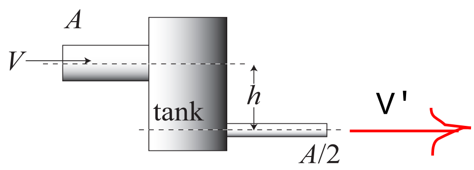
1. There is a hole of radius R in a plate made of a material with (linear) thermal expansion coefficient $\alpha = 3.1 \times 10^{-5}$ K^{-1}. In this hole is a disk of radius exactly R made of another material with (linear) expansion coefficient $\alpha' = 1.7 \times 10^{-5}$ K^{-1}. Now, the temperature is reduced by 100 K.



Is the disk easier to take out of the hole? You must justify your answer. [5]

```
The whole system is cooled, so R shrinks: which shrinks more, the
hole or the disk? The disk has a smaller expansion coefficient than
the hole, so the hole shrinks more. It is now much harder to take the
disk out of the hole 100 K below.
```

2. To a tank is connected two tubes of cross section A and $A/2$, respectively, as illustrated below. The difference between the heights of the centers of the pipes is h, as in the figure. The speed of the input flow from the left is V. The pressures in the pipes are identical. The flow is in a steady state.



V'

Continuity equation

Find V in terms of g and h. [5]

P_A = atmospheric pressure

```
This is a standard Bernoulli question:
          P + (1/2)\rho V^2 + \rho g h = const.
To use this, we need the exit speed of the fluid V':
     AV = (A/2)V', so V' = 2V.
In: P_A + (1/2)\rho V^2 + \rho g h
Out: P_A + (1/2)\rho (2V)^2 + 0
These two formulas must be identical:
     (1/2)\rho V^2 + \rho g h = 2 \rho V^2.
That is, gh = (3/2)V^2, or V = \sqrt{2gh/3}.
```

(**3** on the next page)

[1]edited by Alex Weiss

This doubles the number of particles. That is, n -> 2n

3.

a. In a container of volume 0.2 m^3 is an ideal gas consisting of a chemical species A at temperature $T_0 = 300$ K. Its pressure is P_0. Now, the gas molecules decompose into two chemical species as A $\rightarrow$ B + C. The temperature has become $T = 450$ K, and the final pressure is P. Obtain P/P_0. [5]

```
You need not calculate P_0. P = nRT/V.
initially: T_0 = 300, V = 0.2, P_0 = n_0RT_0/V.
finally: T = 450, V = 0.2, n = 2n_0, P = nRT/V = 2n_0RT/V.
Therefore,
     P/P_0 = 2n_0T/2n_0T_0 = 2(450/300) = 3.
```

b. The ratio of the molecular mass of B and that of C is 3 ($M_B/M_C = 3$). What is the ratio v_B/v_C of the root-mean square velocity v_B of B and v_C of C? [5]

```
Mv_{rms}^2 is proportional to T, so
     v_B is proportional to \sqrt{T/M_B}
     v_C is proportional to \sqrt{T/M_C}.
Therefore,
     v_B/v_C = \sqrt{M_C/M_B} = 1/\sqrt{3} = 0.58.
```

Physics 101 (S11) **Q11D**[1]

Name: ______________ **Section:** __________ **Score:** __________/20

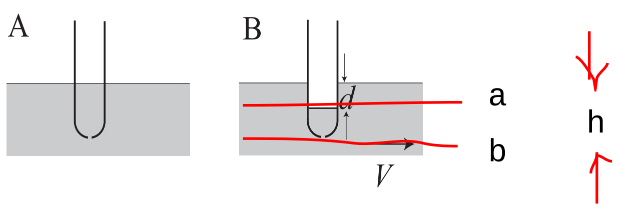
1. There is a block of volume 250 m^3 at $T = 310$ K, made of a material with (linear) thermal expansion coefficient $\alpha = 3.1 \times 10^{-5}\ \mathrm{K}^{-1}$. Now, the temperature is reduced by 100 K. What % is the reduction of the volume of the block? [5]

```
Let \beta be the volume expansion coefficient.  Then,
V(210) = V(310)(1 - \beta x 100).  Thus,
     V(210)/V(310) = 1 - 100\beta
Therefore, the answer must be 10^4 x \beta.  \beta = 3 \alpha, so
 3 x 3.1 x 10^{-5} x 10^4 = 0.93 %.
```

Recall
(1 + a)^3 = 1 + 3a

2. At the bottom of a tube is a small hole. When the tube is immersed in still water (as illustrated in figure A), the height of the surface of the liquid on the inside of the tube coincides with that on the outside. Now, consider a different situation in which the water is moving at a constant speed V (as in figure B). Let us immerse the tube just as in A.

This was conceptually a bit too hard.



P_A = atmospheric pressure

This is Pascal.

Write down the difference d of the heights of the inside and the outside water surfaces of the tube in terms of ρ, V and g. [5]

```
Mr Bernoulli tells us that P + (1/2)\rho V^2 + \rho g h = const.
The easiest way is to imagine the tube is moving at speed V in the
still water. In this way, we can obtain the pressure at the tip with
the use of Pascal's law:
a: P_A + (1/2)\rho V^2 + \rho g (h - d)
b: (P_A+\rho g h) + 0 + 0
These two formulas must be identical:
     P_A + (1/2)\rho V^2 + \rho g (h - d) = (P_A+\rho g h)
or
     d = V^2/2g.
```

(**3** on the next page)

[1] edited by Alex Weiss

17.05 g

3.

a. In a container of volume 0.1 m^3 is 3.1 moles of an ideal gas consisting of a single chemical species at temperature $T = 300$ K. Its pressure is P_0. Now, 17 kg of the same ideal gas is injected into the container without changing its temperature. The pressure has increased by 10 %. What is the molecular mass of the ideal gas? [5]

Let us first write down the problem in terms of formulas:

P_0 = 3.1RT/V (T and V are not changed, so let us not write them explicitly)

1.1P_0 = (3.1 + 17.05/M)RT/V.

Therefore,

1.1 = 1 + 17.05/3.1M.

Therefore, M = 170.5/3.1 = 55 amu.

b. Now, the volume of the container is halved and its temperature is adjusted to be 300 K again. Let K_A be the kinetic energy per molecule prior to halving the volume and let K_B be the kinetic energy after the halving. What is the ratio K_A/K_B? [5]

K is solely determined by T, so the ratio is 1.