\Delta L = \alpha L_0 \Delta T (in the formula sheet)

L = L_0 + \Delta L
= L_0[1 + \alpha L_0 \Delta T]

**Physics 101 (F11)**

**Name:** ____________ **Section:** ________ **Score:** ________/20

**1**. Let $\rho(T)$ be the density of a substance at $T$ (K). It is found that $\rho(600)/\rho(300) =$ ~~1.038.~~ 0.96
What is the linear thermal expansion coefficient $\alpha$ of this substance? [5]

```
\rho(T) = M/V(T) [definition of density is mass/volume]
     V(T) = V(T')[1 + \alpha (T-T')]^3
which is almost equal to
     V(T')[1 + 3\alpha (T-T')].
Hence,
     \rho(T) = M/{V(T')[1 + 3\alpha (T-T')]}
             = \rho(T')/[1 + 3\alpha (T-T')]
which is almost equal to
     \rho(T) = \rho(T')[1 - 3\alpha (T-T')].
This implies
     \rho(600)/\rho(300) = 1-900\alpha = 0.96.
Therefore,
     \alpha = 0.04/900 = 4.44 x 10^{-5}/K.
```

If x is very small, then
(1 + x)^2 = 1 + 2x + x^2 is almost
1 + 2x, because x^2 << x.
(1 + x)^3 is almost 1 + 3x,
Generally,
(1+ x)^d is almost 1 + dx
In particular,
1/(1 + x) is almost 1 - x.

If you do not use the simplification
(1 + x)^d \simeq 1 + dx,
then
you get 4.6 x 10^{-5} /K.

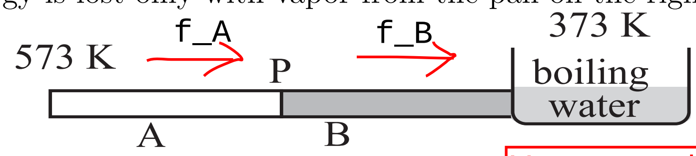
**2**. In a rigid container is an ideal gas consisting of a mixture of equal number of molecules of chemical species B and C at temperature $T_0 = 200$ K. Its pressure is $P_0$. After the completion of the reaction $B + C \to A$ (i.e., after the total number of particles is halved), the temperature goes up to $T = 350$ K and the pressure is $P$. What is $P/P_0$? [5]

Only the number of molecules matter.

```
Before
     P_0V = nRT_0  or P_0 = (nR/V)T_0
After
     PV = (n/2)RT  or P = (nR/V)T/2
Therefore,
     P/P_0 = T/2T_0 = 350/400 = 0.875.
```

(**3** on the next page)

**3**. Two heat conducting rods A and B are connected at P and attached to a container C with boiling water at 100°C as shown in the figure. The left end of rod A is maintained at 573 K. The thermal conductivity of rod A is $\kappa_A$ and that of rod B is $\kappa_B$. Their lengths and cross sections are the same. You may assume that the temperatures are all constant. (You assume there is no exchange of heat between the rods and its surroundings except at their ends. Thermal energy is lost only with vapor from the pan on the right.)




(a) $\kappa_A/\kappa_B = 3$. What is the temperature at P? [5]

```
The heat flow through A and that through B must be identical.
Through A:
     f_A = \kappa_A(A/L)(573 - T),
Through B:
     f_B = \kappa_B(A/L)(T - 373).
Since f_A = f_B
     kappa_A(573 - T) = kappa_B(T - 373)
or
     3(573 - T) = T - 373, i.e., 4T = 3x573 + 373 = 2092.
T = 523 K.
```

You must subtract low from high temp.

The formula sheet gives Q = kA\Delta T t/L. This means that the heat flow rate f is
f = Q/t = k A \Delta T/ L.

(b) In 10 minutes 120 g of water evaporates. What is the rate of energy flow through the the cross section at P (in W or in kW)? The latent heat of evaporation of water at 373K is 2260 kJ/kg. [5]

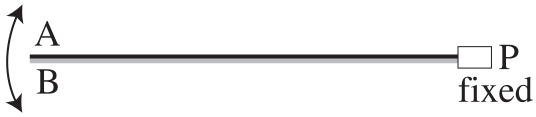
```
The required heat energy is 2260 x 0.12 kJ in 600s, so the power
transferred is
P = 2260 x 0.12/600 = 0.452 kW.
```

**Physics 101 (F11)** **Q11B**

\a = \alpha

**Name:** ______________ **Section:** ________ **Score:** ________/20

**1**. A bimetallic strip (bimetal) made of two metal strips A and B is horizontally fixed at its one end P as shown in the following figure. The linear thermal expansion coefficient of metal A is $\alpha_A = 1.4 \times 10^{-5}$ K$^{-1}$, and that of metal B is $\alpha_B = 2.3 \times 10^{-5}$ K$^{-1}$. The figure shows the state of the bimetal at 310 K. At 290 K which way does it bend, upward or downward? You must justify your answer. [5]




```
Since \a_A < \a_B, B is more sensitive to the temperature change.
When the temperature is lowered, B shrinks more. Therefore the strip
bend downward.
```

<K> = 3k_BT/2 is determined solely by T.

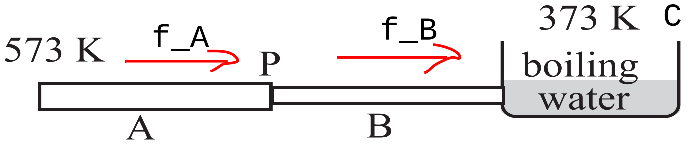
**2**. In a container is an ideal gas mixture consisting of equal numbers of molecules of chemical species B and C at temperature $T_0 = 200$ K. The ratio $v_B/v_C = 1.3$, where $v_B$ (resp., $v_C$) is the root-mean-square velocity of molecule B (resp., C). What is the ratio of the molecular weights $M_B/M_C$, where $M_B$ (resp., $M_C$) is the molecular mass of molecule $B$ (resp., C). [5]

```
Since the average kinetic energy is solely determined by T, both the
molecules have the same average kinetic energy.
     We know <K> = (1/2)m<v^2>, so
     M_Bv_B^2 = M_C v_C^2 or
     M_B/M_C = v_C^2/v_B^2 = (1/1.3)^2 = 0.592.

Faster molecules must be lighter at the same temperature.
```

(**3** ~~and 4~~ on the next page)

**3**. Two heat conducting rods A and B are connected at P and attached to a container C with boiling water at 100°C as shown in the figure. The left end of rod A is maintained at 300°C. Both the rods are made of the same material and of the same lengths, but the cross section of A is twice as large as that of B. (We assume there is no exchange of heat between the system under consideration and its surroundings except at the left end of A and due to evaporation from the pan. Also you may assume that the rods are far longer than their widths.)




(a) What is the temperature at P (in the stationary state)? [5]

The heat flux through A and that through B must be identical.
Through A
    f_A = \kappa A(573 - T)/L.
Through B
    f_B = \kappa (A/2)(T - 373)/L.

High T - Low T

Since f_A = f_B in the steady state
    573 - T = (T - 373)/2.
That is,
    3T = 2 x 573 + 373 = 1519 or T = 506 K (233 degree C).
This T must be closer to 573 than to 373.

Since A conducts heat better than B, the temperature at P should be closer to the A end.

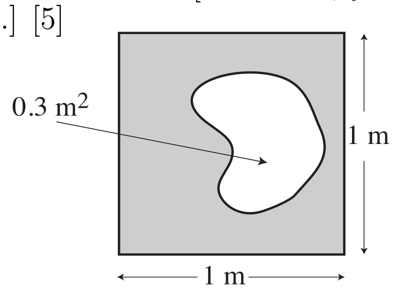
(b) In 3 minutes 20 g of water evaporates under the current condition. If the temperature at the left end of A is changed to 473 K how long does it take to evaporate 20 g (i.e., the same amount) of water? [5]

Since the temperature difference is halved (from 200 K to 100 K), the overall heat transfer rate should be halved. Then, the needed time should be doubled. 6 min.

**Physics 101 (F11)** **Q11C**

**Name:** ____________________ **Section:** __________ **Score:** __________/20

1. A square is made of a material whose thermal expansion coefficient is $\alpha = 3.8 \times 10^{-4}$ $K^{-1}$. The square has a hole as shown in the figure below. Its area is 0.3 $m^2$ at $T = 210$ K. What is the area of the hole at $T = 450$ K? [Of course, you must assume that the material stays solid at this temperature.] [5]




(1 + x)^2 = 1 + 2x is very accurate, if x is very small.

```
The expansion of the `window' is exactly the same as the material just
filling the window.
Area at temperature $T$ and that at temperature $T'$ can be related as
     A(T) = A(T') [1 + \alpha(T - T')]^2,
which is almost identical to
           A(T') [1 + 2\alpha(T - T')].
Therefore,
     0.3[1 + 2 x 3.8 x 10^{-4} x 240] = 0.3 x 1.182 = 0.355 m^2.
```

2. A large enough ice chest has 1 kg of liquid water and 0.5 kg of ice and is at 0°C initially. Into this ice chest you put 7 soda cans at 25 °C whose heat capacity is 980 J/K per can. When the temperature reaches a new equilibrium, is there ice remaining in the ice chest? The latent heat of fusion of ice is 333 kJ/kg. You may ignore the heat capacity of the ice chest. [5]

```
If ice is to remain, the soda cans and the water must be at 0 deg C.
The required heat we must remove from the cans to cool them to 0 deg C
is
     C \Delta T = (7 x 980) x 25 = 171500 J = 171.5 kJ.
This corresponds to 171.5/333 = 0.515 kg of ice to melt.  We have
only 0.5 kg of ice.  Hence, no ice remains.

By the way, what is the final temperature?
Let T be the final temperature. (7 x 980) x (25 - T) of thermal
energy is removed from the cans and  used to melt the ice
(0.5 x 333000 J needed) and to raise the temperature of 1.5 kg
of 0 deg C water to T deg C.  The specific heat of water is
4200 J/kg.K, so
     (7 x 980) x (25 - T) = 0.5 x 333000 + 4200x1.5T
or
     171500 - 6860T = 166500 + 6300T
     5000 = 560T  that is T = 8.9 deg C.
```

Heat capacity

(3 ~~and 4~~ on the next page)

(1/2)m<v> ^2= (3k_B/2)T, so
T is proportional to <v>^2

**3**. The molar heat capacity of an ideal gas is $5R/2$. One mole of this gas is in a rigid container of volume $V$ and is initially at 190 K and pressure 2300 Pa. Now, the heat is added and the pressure reaches 8500 Pa.

(a) Is the root-mean square velocity of the gas more than doubled? You must justify your answer. [5]

```
We must compare T.  To double v, we must quadruple T.
Initially,
     PV= nRT means 2300V = R x 190
Finally,
     8500V = R x T.
You should have realized that P is proportional to T.
Since the pressure is not quadrupled, the answer is NO.

The final temperature is
     190/2300 = T/8500, so T = 702 K (< 4 x 190 = 760 K).
```

(b) What is the total amount of heat added to the ideal gas to raise its temperature? [5]

```
The needed heat is Q = C \Delta T = (5R/2)\Delta T, since we have 1
mole of gas.
     According to the above calculation \Delta T = 702 - 190 = 512 K.
Hence,
     Q = (5 x 8.31/2)512 = 10637 = 10.6 kJ.
```

(1 + x)^d is very close to 1 + dx if x is very small (if x^2 << x)

**Name:** ____________ **Section:** ________ **Score:** ________/20

**1**. At $T = 250$ K a cubic container of volume 0.8 $\mathrm{m}^3$ made of a metal of [illegible] thermal expansion coefficient $\alpha = 3.5 \times 10^{-5}$ $\mathrm{K}^{-1}$ contains brimful of liquid whose [illegible] thermal expansion coefficient is $\alpha = 4.5 \times 10^{-5}$ $\mathrm{K}^{-1}$. Now the temperature of the system is raised to 330 K. What is the volume of the spilt liquid (at 330 K)? [5]

```
The relation between the volume at 250 K and that at 330 K is
     V(330) = V(250)[1 + \alpha (330 - 250)]^3
which is almost identical to
            = V(250)[1 + 3\alpha (330 - 250)].
Now, we are interested in the volume increase difference between the
container and the liquid.
Liquid increase = 0.8[3 x 4.5 x 10^{-5} x 80] = 0.00864 m^3
Container increase = 8[3 x 3.5 x 10^{-5} x 80] = 0.00672 m^3
Thus, 0.00192 m^3 of liquid spills.
```

This is increase.

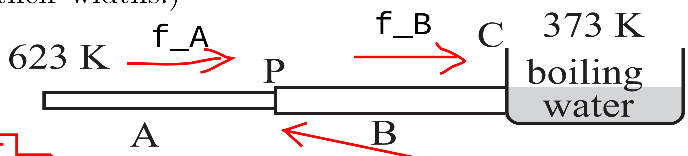
<K> = (3k_B/2)T

**2**. In a rigid container is an ideal gas of pressure $P_0$. What is the pressure $P$ of the ideal gas, if you wish to double the average kinetic energy of the gas molecules? Give $P/P_0$. you may assume that the container volume does not change. [5]

```
P_0V = nRT.  To double the kinetic energy T must be doubled.
Hence, P/P_0 = 2.
```

(**3** on the next page)

3. Two heat conducting rods A and B are connected at P and attached to a container C with boiling water at 100°C as shown in the figure. The left end of rod A is maintained at 350 °C. Both the rods are made of the same material and of the same lengths, but the cross section of B is twice as large as that of A. (We assume there is no exchange of heat between the system under consideration and its surroundings. Also you may assume that the rods are far longer than their widths.)




High T - Low T

The temperature here should be closer to 373 than to 623 K

(a) What is the temperature at P (in the stationary state)? [5]

```
The heat flow rate through A and that through B must be identical.
Through A
     f_A = \kappa (A/2)(623 - T)/L,
Through B
     f_B = \kappa A(T - 373)/L.
Therefore, f_A = f_B implies
     (623 - T)/2 = (T - 373)
or
     3T = 623 + 2 x 373 = 1369, so T = 456 K (183 deg C)
```

The formula sheet tells us
  Q = kA\Delta Tt/L,
so heat flow rate f is
  f = Q/t = kA\Delta/L

(b) It takes 3 minutes to evaporate 10 g of water. Suppose the thickness of the rod A is just the same as B (that is, both have the same cross sections). How many minutes does it take 10g of water to evaporate? [Hint: What is the temperature at P now? Then, concentrate your attention to the heat through B.] [5]

```
Now the temperature at P = 498 K.  Therefore, f_B is increased from
\kappa A(456 - 373)/L to \kappa A(498 - 373)/L.  That is, the ratio of
energy flow is
 (498 - 373)/(456 - 373) = 125/83 = 1.5,
or now it is 1.5 times as large as the previous case.  Hence, 2 minutes
must be enough.
```