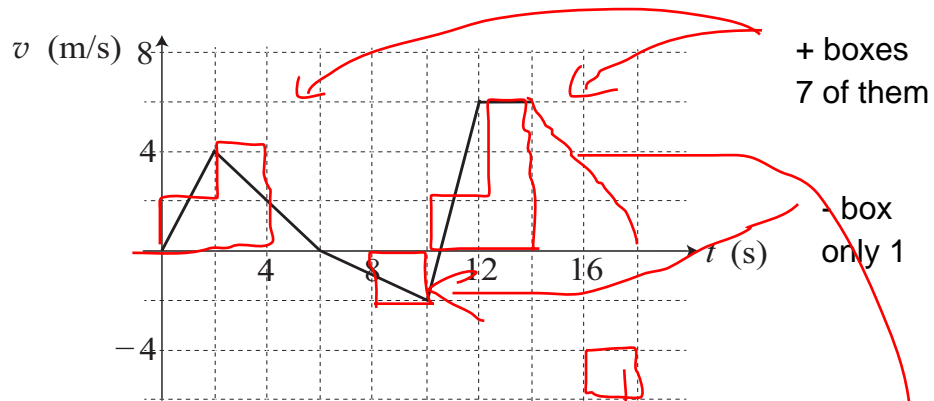


Name: _____ Section: _____ Score: _____/20

1. A cart of mass 2.5 kg is moving along the x -axis. Its velocity (x -velocity) as a function of time is graphed below.



(a) What is the average velocity of the cart between time 0 and 14 seconds? [4]

One square in the graph = 2 m/s times 2 s = 4 m.

We need the signed area = 7 - 1 = 6 boxes = 24 m. Therefore, the average velocity is

$$24/14 = 12/7 = 1.7 \text{ m/s.}$$

(b) What is the maximum magnitude of the (total) force acting on the cart before 14 s? [4]

Total force = mass times acceleration, so we need the largest magnitude of the acceleration.

The acceleration is given by the slope of the time-velocity graph.

The steepest portion is between 10 and 12 s, and the slope is (+) 4 m/s².

Hence, the magnitude of the force is 2.5 times 4 = 10 N.

(c) Immediately after 14 s a brake is applied, and the cart experiences a constant acceleration of -1.5 m/s^2 . Continue the velocity graph beyond 14 s until the cart comes to a halt. [2]

The cart comes to a halt at 18 s.

(2 on the next page)

2. A toy rocket is fired vertically with zero initial velocity from the ground with a constant acceleration a (m/s^2). After 2 s the fuel is exhausted. Still the rocket continues to go up (ignore air resistance; only gravity acts on the rocket). After 3 s (i.e, 5 s after launching), the rocket reaches its highest point.

(a) What is the acceleration a ? [5]

For the initial 2 seconds, the acceleration is a , so the upward velocity at time 2 s is $2a$.

Since the fuel is gone, after this, the rocket problem is just the ball thrown upward with the initial velocity $2a$.

At the highest point the vertical velocity must vanish. $v(t) = v(0) + (-g)t$, since the acceleration is downward due to gravity. That is, $0 = 2a - 3g$. Therefore, $a = 3g/2 = 14.7 \text{ m/s}^2$.

(b) What is the vertical distance the rocket traverses after exhausting its fuel until it reaches the highest point? If you are not sure about your answer to (a), you may use a as a symbol in your answer. [5]

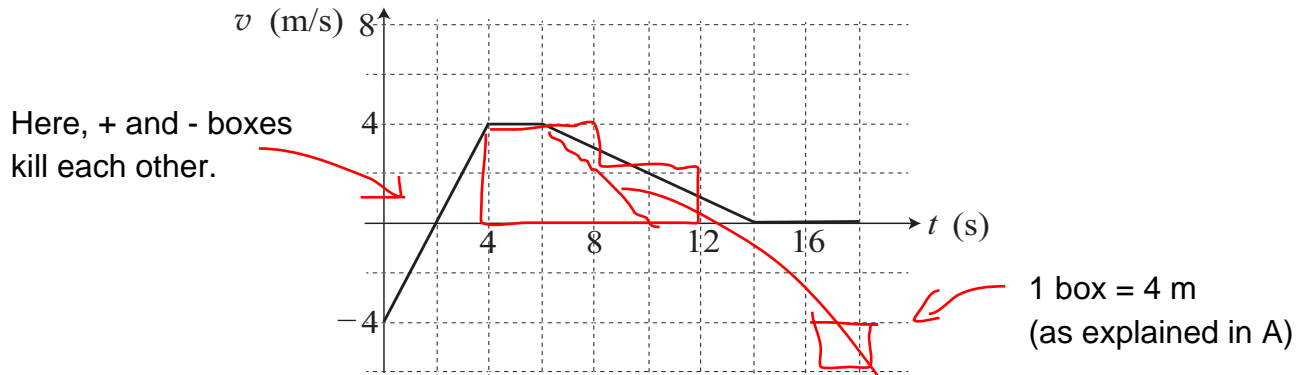
The cleverest solution: Let's play the movie backward. It is the free fall from the highest point to the height where the rocket exhausted all its fuel. This is without any initial velocity, because it comes to a halt at the highest point (as noted in (a)).

Therefore, $h = (1/2)g t^2 = (1/2)g 3^2 = 4.5 g = 44.1 \text{ m}$.

You could use $x = x(0) + v_0 t + (1/2)(-g)t^2$, with $v_0 = 2a = 3g$, $x(0) = 0$, $x = h$, and $t = 3$.

Name: _____ Section: _____ Score: _____/20

1. A cart of mass 1.7 kg is moving along the x -axis. Its initial position is $x = 0$ (m). Its velocity (x -velocity) as a function of time is graphed below.



(a) The cart eventually stops. What is its final position (x -coordinate)? [4]

Total 6 boxes = 24 m. This is the total displacement in the positive x direction

The initial position is 0, so $x = 24$ (m).

(b) What is the maximum magnitude of the (total) force acting on the cart? [4]

The max force is due to the max acceleration.

The acceleration is given by the slope of the t - v curve: $a = 2 \text{ m/s}^2$.

$F = ma = 1.7 \times 2 = 3.4 \text{ N}$.

(c) If the magnitude of the acceleration (actually, the deceleration) is doubled beyond 6 s, when does the cart come to a halt? (Answer by drawing (i.e., revising) the time velocity graph beyond 6 s.) [2]

At 10 s.

(2 on the next page)

2. On a planet is a tower of height h (m). A ball is thrown vertically upward with an initial speed of 4 m/s from the top of the tower. The ball reaches the highest point after 6 s, and then it lands on the ground 9 s later (i.e., 15 s after the ball is thrown upward).

(a) What is the acceleration of gravity g_0 of this planet? [5]

At the highest point the vertical velocity vanishes.

$$v(t) = v(0) + (-g_0) t,$$

where $v(t) = 0$, $v(0) = 4$, $t = 6$, so $6g_0 = 4$ or $g_0 = 2/3 \text{ m/s}^2$.

(b) What is the height h of the tower? If you are not sure about your answer to (a), use g_0 as a symbol in your answer. [5]

We have only to pay attention to the motion after reaching the highest point; you need not worry at all how the ball reaches there!

This is nothing but a free fall problem with 0 initial velocity. It takes 9 seconds to fall from the highest point. Therefore the height H of the highest point is

$$H = (1/2)g_0 \times 9^2 = 81/3 = 27 \text{ m}$$

To fall to the height of the tower, it takes 6 s (Play the movie backward!), so

$$H - h = (1/2) g_0 \times 6^2 = 36/3 = 12 \text{ m.}$$

Thus, $h = 15 \text{ m}$.

Perhaps you do not like this solution. You can use the 'incantation'

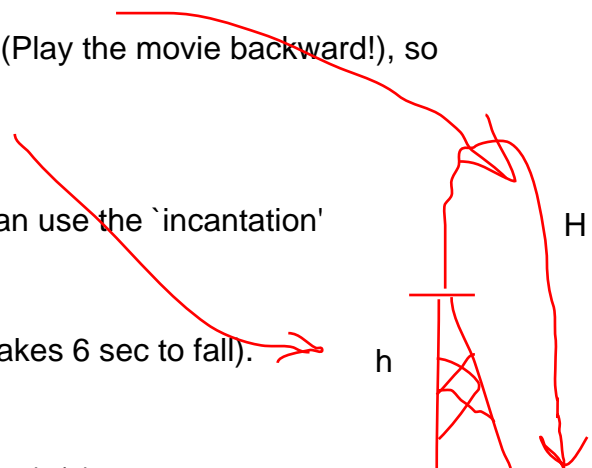
$$v^2 = v(0)^2 + 2a \Delta x.$$

Initial state: $x_0 = h$, $v_0 = 4$.

Final state: $x = 0$, $v = 9 g_0 = 6$ (because it takes 6 sec to fall).

That is $\Delta x = -h$, $a = -g_0 = -2/3$.

Thus, we get: $6^2 = 4^2 + 2(2/3)h$, or $h = 20 \times (3/4) = 15 \text{ m}$.



Name: _____ Section: _____ Score: _____/20

1. The vertical velocity (upward positive) of an elevator is graphed in the following. The height (position) of the elevator floor is initially $z = 0$ (m).



1 box = 2 m

The previous version erroneously said this was 4 m, so all the answers were doubled.

(a) What is the height of the elevator floor at the first stop? [4]

4 boxes = 4 times 2 = 8 m.

(b) What is the average velocity of the elevator between time 0 and 12 s? [4]

We need the total displacement.

We need the signed area = 4 (+)boxes + 3(-)boxes = 1 box = 2 m

Therefore, $2/12 = 1/6$ m/s is the average velocity.

(c) Unfortunately, the elevator cable snaps at time 2 s and it falls freely. Sketch the velocity of the elevator in this unfortunate case as a function of time after 2 s in the graph (approximately quantitatively). [2]

This must be a free fall, so the slope must of the t-v curve must be -9.8 m/s^2

(2 on the next page)

2. A ball is thrown down from the top of a building of height H (m) with an initial speed of v_0 (m/s). At the half height $H/2$ of the building, the speed of the ball is 25 m/s. When the ball reaches the ground, the speed of the ball is 30 m/s.

(a) What is the height H of the building? [5]

minus, because downward

This is the problem for ' $v^2 = v_0^2 + 2at$.'

$$30^2 = 25^2 + 2(-g)(-H/2)$$

minus because the initial height is higher.

Here, we use the last part of the movement from $H/2$ to the ground. $H = (900 - 625)/9.8 = 28.1$ m.

(b) What is the initial speed? If you are not sure about your answer to (a), you may use H as a symbol in your answer. [5]

We apply the same formula to the early part of the movement from H to $H/2$:

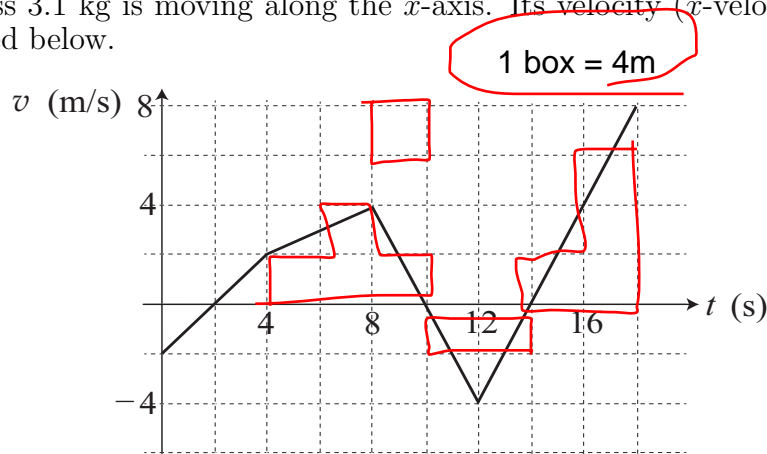
$$25^2 = v_0^2 + 2(-g)(-H/2)$$

$$\text{so } v_0^2 = 25^2 - gH = 2 \times 25^2 - 30^2 = 1250 - 900 = 350$$

$$\text{That is } v_0 = \sqrt{350} = 18.7 \text{ m/s}$$

Name: _____ Section: _____ Score: _____/20

1. A cart of mass 3.1 kg is moving along the x -axis. Its velocity (x -velocity) as a function of time is graphed below.



(a) What is the average velocity of the cart between 0 and 18 s? [4]

We need the signed area 8 plus boxes and 2 minus boxes = 6 boxes
 = 24 m.
 Hence, the average velocity is $24/18 = 4/3$ m/s.

(b) What is the maximum magnitude of the (total) force acting on the cart? [4]

The total force - mass times the acceleration.
 The acceleration is the slope of the tv-curve.
 We need the max of the magnitude of the slope. This occurs between 8 and 12 seconds.
 $a_{\max} = 2 \text{ m/s}^2$. $F = 3.1 \text{ times } 2 = 6 \text{ N}$.

(c) When does the cart return to the starting point (its position at $t = 0$)? [2]

At $t = 4$ seconds.

(2 on the next page)

2. From the top of a tower of height H , a ball is vertically thrown upward with an initial speed 21 m/s. The ball reaches the ground 2 s after returning to the height of the tower (i.e., the initial position).

(a) What is the height H of the tower? [5]

When the height is the same as the initial position, the speed is the same; the velocity flips its sign. That is 21 m/s downward (- 21 m/s upward).

Apply

$$x(t) = x(0) + v_0 t + (1/2)at^2$$

with $x(0) = H$, $x(t) = 0$, $t = 2$, $a = -g$ (because our usual convention is upward +), $v_0 = -21$.

Hence,

$$0 = H - 21 \times 2 + (1/2)(-9.8)2^2$$

or

$$H = 42 + 2 \times 9.8 = 61.6 \text{ m.}$$

(b) What is the total traveling time of the ball since it is initially thrown upward? [5]

The total time must be twice of the time needed to reach the highest point + 2 seconds.

Notice that the needed time to reach the highest point and the time needed to return from there to the initial height are identical.

The vertical speed must vanish at the highest point! Hence, $0 = 21 - 9.8 t$ or $21/9.8 = 2.14 \text{ s}$ is the needed time.

The total time is $2.14 \times 2 + 2 = 6.3 \text{ s}$.