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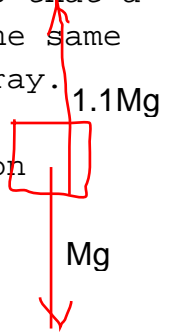
1. A person is on a scale in an elevator. The elevator accelerates and the reading on the scale is 10% larger than it was when the elevator was stationary.

(a) What is the acceleration of the elevator? Pay attention to its sign (the sign convention is upward +). [5]

Let M be the mass of the person. The reading of the scale is $1.1M$.

A scale is a device to measure a force. Its reading X means that a force of Xg is applied to the scale tray. This force has the same magnitude as the normal force acting on the object on the tray.

The free-body diagram is as shown in the right. The equation of motion for is (a is plus when upward)



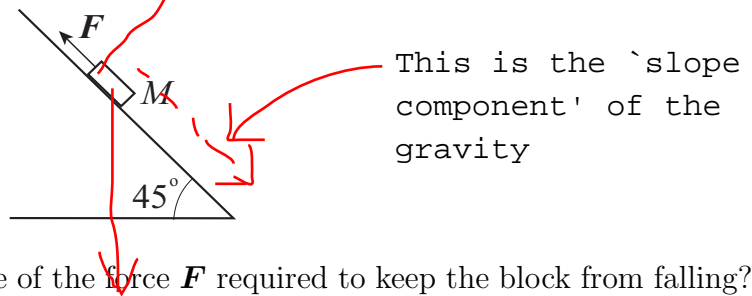
$$Ma = 1.1Mg - Mg \quad \text{hence} \quad a = 0.1 g.$$

(b) After the constant acceleration is over, the reading of the scale returns to the reading when the elevator is stationary. Has the speed of the elevator increased, decreased or remained unchanged? You must justify your answer. [5]

Since 'acceleration' tells only the change of velocity, not the change of speed = $|\text{velocity}|$, we cannot answer this question. If the elevator was going up, then positive a means increase of speed. However, if the elevator was going down, then since the z -component of the velocity is negative, its increase could reduce the speed. Anything is possible.

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2. On an inclined surface with the coefficient $\mu_s = 0.5$ of static friction and the coefficient of kinetic friction $\mu_k = 0.3$ is a block of mass M kg as illustrated in the following figure.



(a) What is the minimum magnitude of the force F required to keep the block from falling? Give the value of $|F|/Mg$, where g is the acceleration due to gravity. [5]

Since the normal force is $N = Mg \cos 45 = Mg/\sqrt{2}$, the MAXIMUM POSSIBLE static friction is $\mu_s N = 0.3 Mg/\sqrt{2} = 0.212Mg$.

The equation of motion along the slope (downward +) reads

$$\begin{aligned} 0 &= -F + Mg/\sqrt{2} - \text{friction} = (0.707 - 0.212)Mg \\ &= -F + 0.495Mg. \end{aligned}$$

That is, $F/Mg = 0.495$.

(b) What is the magnitude of the acceleration a of the block if the force F is removed? [5]

$$Ma = Mg/\sqrt{2} - \text{friction}$$

Here, 'friction' is the kinetic friction $= \mu_k N$ (magnitude).

Therefore,

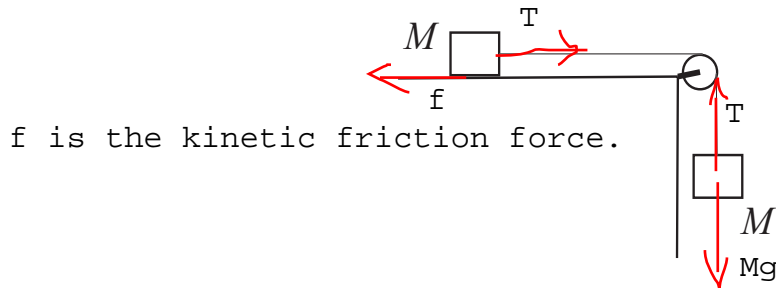
$$Ma = Mg/\sqrt{2} - \mu_k Mg/\sqrt{2} = Mg(1-0.3)/\sqrt{2}.$$

That is,

$$a = 0.7g/\sqrt{2} = 0.495g = 4.85 \text{ m/s}^2.$$

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1. There are two identical blocks of mass M . One is on the rough and horizontal table and is connected to the other mass via a massless string through a massless and frictionless pulley as shown below.



f is the kinetic friction force.

Be careful; the block is going down faster and faster.

(a) The magnitude of the acceleration a of the hanging mass is $a = g/4$, where g is the acceleration due to gravity. Let the tension in the string be T . What is T/Mg ? [5]

From the free-body diagram above,

$$Ma = T - Mg,$$

where the sign convention is + upward, so $a = -g/4$. Hence,

$$T = M(g + a) = (3/4)Mg.$$

(b) What is the coefficient μ_k of kinetic friction between the mass on the table and the table surface? [2]

The friction is kinetic: $f = \mu_k N = \mu_k Mg$. From the free-body diagram for the mass on the table (+ sign to the left)

$$Ma = f - T = (\mu_k - 3/4)Mg = -Mg/4,$$

so $\mu_k = 1/2$.

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2. An elevator is initially going down at a speed 3m/s, and then decelerates uniformly in 2 seconds and comes to a halt. In the elevator stands a person of mass M (kg) on a scale.

(a) The scale reads M' (kg) during the deceleration. What is the ratio M'/M ? [For the sign convention, choose the upward direction to be positive. Hint: Is the acceleration positive or negative?] [5]

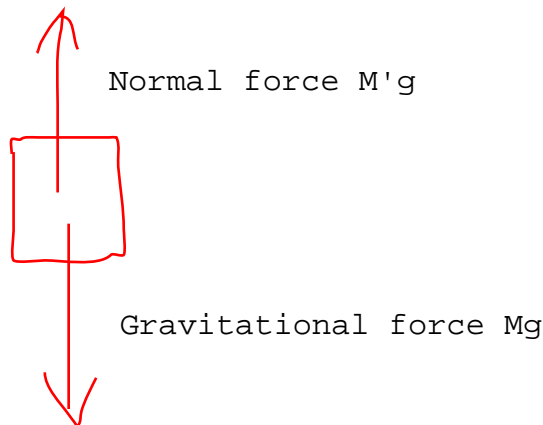
Let us choose the sign convention that upward is +.

Then, the initial velocity is -3 m/s, which goes to 0, so $a = 3/2 = 1.5 \text{ m/s}^2$ (it is POSITIVE).

The normal force acting on the person is $M'g$ (upward).

Therefore,

$$Ma = M'g - Mg \quad \text{or} \quad M'/M = 1 + a/g = 1 + 0.15 = 1.15.$$

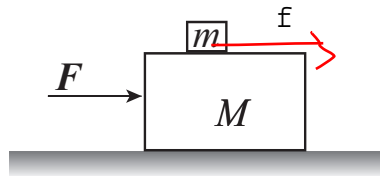


(b) Unfortunately, the cable of the elevator snaps. What is the reading of the scale now? You must justify your answer. [5]

Now, the person's acceleration is $-g$, so $M' = 0$ according to the equation of motion we wrote down above. 0.

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1. On a frictionless horizontal floor is a block of mass M , on which sits another mass of m as shown below. The value of m is actually one half of M (i.e., $m = M/2$). The top surface of mass M is horizontal but not smooth, and the coefficient of static friction between m and M is $\mu_s = 0.3$.



(a) When a horizontal force F is applied, the two masses move together with an acceleration of 1.0 m/s^2 . Let the magnitude of the horizontal friction force acting on m be f . What is f/m ($= 2f/M$)? [5]

Realize that the top block can move only thanks to the friction force f . $ma = f$, $a = 1.0$. Therefore, obviously, $f/m = 1.0 \text{ m/s}^2$.

(b) What is the minimum magnitude of the force F required for the mass m to start to slide? You may assume $M = 12 \text{ kg}$ (so $m = 6 \text{ kg}$). [(a) is a hint.] [5]

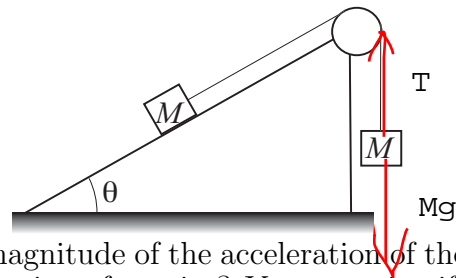
Since f is a static friction force, it cannot exceed $\mu_s mg$, where mg is the normal force acting on the block of mass m . Therefore, according to $ma = f$, a cannot exceed $\mu_s mg/m = \mu_s g$.

Up to this critical acceleration, two blocks move together, so they must have the same accelerations. Hence,

$$F = (M+m) \mu_s g = 0.3 \times 18 \text{ g} = 52.9 \text{ m/s}^2.$$

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2. On a frictionless slope that makes an unknown angle θ with the horizontal is a block of mass M , which is connected to another identical block of mass M with a massless string through a massless and frictionless pulley as illustrated below.



(a) For some positive θ can the magnitude of the acceleration of the hanging mass be smaller than $g/10$, where g is the acceleration of gravity? You must justify your answer. [5]

Think of an extreme case: $\theta = 90$ degrees. Then, the blocks cannot move $a = 0$, which is definitely smaller than $g/10$.

(b) Suppose the acceleration is $0.1g$. What is the ratio T/Mg , where T is the tension in the string? [5]

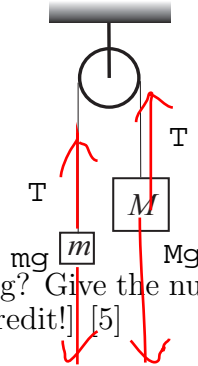
Let us choose upward to be positive (our sign convention).

$$M(-0.1g) = T - Mg.$$

Therefore, $0.9Mg = T$, or $T/Mg = 0.9$.

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1. Two masses m and M are connected with a massless string and hang from a massless and frictionless pulley as illustrated below. When the masses are gently released, the acceleration of M is $g/3$ downward, where g is the acceleration due to gravity.



(a) What is the tension T in the string? Give the number T/Mg . [Hint: Pay close attention to the signs. As usual, no work, no credit!] [5]

From the free-body diagram, we get (Let us choose 'upward +' convention)

$$M(-g/3) = T - Mg$$

Therefore,

$$T/Mg = 2/3.$$

(b) Find the ratio m/M . [5]

The equation of motion for m is

$$m(g/3) = T - mg$$

From this we get

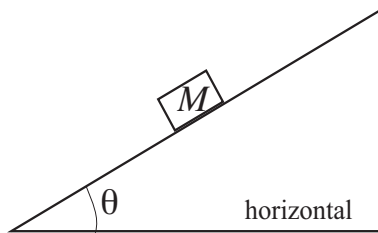
$$(4/3)mg = (2/3)Mg.$$

Therefore,

$$m/M = 1/2.$$

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2. On a rough slope that makes an angle θ with the horizontal is a block of mass M as illustrated below.



(a) When the angle is θ (degrees), the block starts to slide in a lab on earth. What will happen if the same experiment is performed in a lab on the moon? [5]

Let us choose the climbing direction to be positive.

The statement of the problem implies that the maximum possible static friction barely balances with the gravity

The magnitude of the normal force is $Mg \cos \theta$.

The magnitude of the gravity along the slope is $Mg \sin \theta$.

Therefore,

$$0 = \mu_s Mg \cos \theta - Mg \sin \theta, \text{ so } g \text{ does not matter. The same.}$$

(b) When $\theta = 30$ degrees, the block does not slide. What is the magnitude f of the friction force acting on the block on the earth? Find f/Mg . [5]

'Does not slide' means that the friction force is not yet maximum. So we cannot use $f = \mu_s N$. f is determined by the balance condition with the gravity. Therefore,

$$f = Mg \sin \theta = Mg/2$$

$$\text{or } f/Mg = 1/2.$$