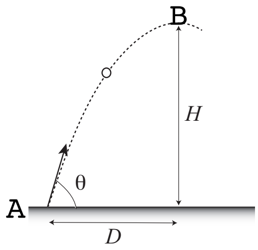
Name: ____________ **Section:** ________ **Score:** ________/20

1. A ball is shot (by a pitching machine) from horizontal ground. It reaches the highest point after 3.2 seconds.



(a) What is the height H of the highest point? [5]

```
(0) The movement from A to B is the movement from B to A `played
backward' in time.
(1) x and y movements can be decoupled.

Therefore, for the y-component, B to A is free fall with zero initial
velocity.  Hence,
        H = (1/2)gt^2 = 4.9(3.2)^2 = 50.176 m.
```

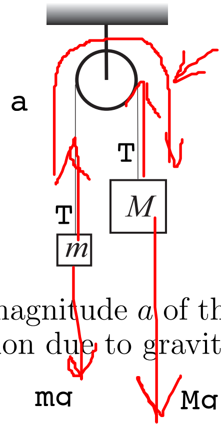
(b) The highest point is exactly above the point which is $D = 100$ m horizontally away from the starting point as in the figure. What is the initial angle θ? [If you are not sure about your answer to (a), find $\tan\theta$ in terms of H and D.] [Hint. Try to obtain the x-component v_x and y-component v_y of the initial velocity in terms of D and H, respectively.] [5]

```
If the initial velocity is V = (v_x, v_y), then
    tan \theta = v_x/v_y,
so we should compute these components.
v_x: the motion along the x-axis is without acceleration, so
    D/t = v_x
v_y: after t sec, the v_y-component vanishes when the ball reaches
    the highest point.  Therefore, 0 = v_y -gt, or v_y = gt.

Therefore,
    tan \theta = gt^2/D= 2H/D = 1, so \theta = 45 degrees.
```

(**2** on the next page)

2. Two masses m and M are connected with a massless string and hang from a massless and frictionless pulley as illustrated below.



Let us choose the positive direction as in the figure along the string.

(a) Suppose $m = M/2$. What is the magnitude a of the acceleration of the blocks? Give the value of a/g, where g is the acceleration due to gravity. [5]

```
You must be able to draw the free-body diagram.
The equation of motion for
m: ma = T – mg  (upward is positive)
M: Ma = Mg – T  (downward is positive).
Hence,
    (m + M)a = (M – m)g or a/g =(M-m)/(M+m) = 1/3.
```

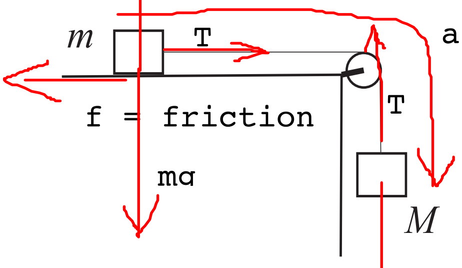
(b) Suppose M is much larger than m (say, $M = 10^4 m$). What is the magnitude a of the acceleration of m? [5]

```
It's nothing but free fall of M, but m follows M, so g.

Or, from the above computation a/g -> 1.
```

Name: ______________ **Section:** __________ **Score:** __________/20

1. A block of mass m is on a rough and horizontal table with coefficient of kinetic friction $\mu_k = 0.2$ and is connected to another block of mass M via a massless string through a massless and frictionless pulley as shown below.



Let us choose the positive direction along the string as illustrated here (you can choose the opposite direction).

(a) Suppose $m = M/2$. What is the magnitude of the acceleration a of the block of mass m on the table? Give the ratio a/g, where g is the acceleration due to gravity. [5]

```
You must be able to draw the free-body diagrams.
Let us write down the initial
m: ma = T - f,  you know f = \mu_k mg      normal force
M: Ma = Mg - T.
Therefore,
    (m + M)a = Mg - \mu_k mg
so
    a/g = (M - m \mu_k)/(M+m) = (1- 0.5x0.2)/1.5 = 0.6.
```

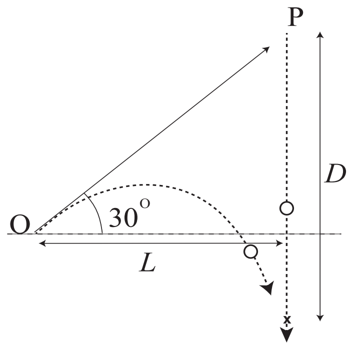
(b) Suppose M is much larger than m (say, $M = 10^4 m$). What is the ratio a/g just discussed? You must justify your answer. [2]

```
It is just a free fall of M, so g (m must follow M).

Or, in the formula obtained above, take M/m -> infinity.  We
get a/g -> 1.
```

(**2** on the next page)

2. At the moment when a ball is gently released from P, you shoot another ball from O aiming at the ball at P. The point P is exactly above the point that is $L = 3$ m horizontally away from you as illustrated. The line connecting O and P makes an angle of 30° with the horizontal.



(a) The two balls collide at the cross-mark, which is $D = 4.9$ m below P. Find the initial speed V. [Hint. First, try to calculate the x-component V_x of the initial velocity.] [5]

```
Let us pay attention to the time t when the collision occurs.
    D = (1/2) gt^2.
The x-motion and the y-motion are totally decoupled, so
    V_x t = L.
Therefore,
    V_x^2 = L^2/t^2 = gL^2/2D or V_x = \sqrt{g/2D} L = 3 m/s.

    V = V_x/cos 30 = 2 \sqrt{3} = 3.6 m/s.
```

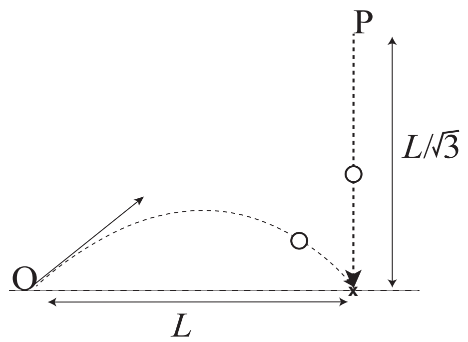
(b) Suppose you do the same experiment on a planet whose acceleration of gravity is one half that on earth (i.e., $g/2$). To keep the L and D, what is the new initial speed V'? Obtain V'/V. [5]

```
V_x = \sqrt{g/2D} immediately tells you that V' = V/\sqrt{2}.

If you do not wish to use this result, go to the basic, again:
D = (1/2)gt^2, V_x = L/t, so t -> \sqrt{2} t, and V 0> V/\sqrt{2}.
```

Name: ____________________ **Section:** __________ **Score:** __________/20

1. At the moment when a ball is gently released from P, you throw another ball from O aiming at the ball at P. The point is exactly above the point that is $L = 5$ m horizontally away from you as illustrated. The line, which is the direction of the initial velocity, connecting O and P, makes an angle of 30° with the horizontal.



(a) Obtain the x-component of the initial velocity of the ball you throw. [5]

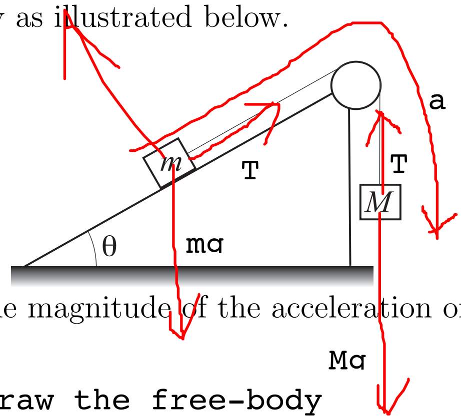
```
Pay attention to the time t when the collision occurs.
L/\sqrt{3} = (1/2) gt^2.
V_x = L/t, so V_x^2 = L^2/t^2 = (\sqrt{3}/2)gL = 42.4, or
V_x = 6.51 m/s.
```

(b) What is the speed of the ball you throw when it hits the other ball? [5]

```
This is the same as the initial speed, so
V = V_x/\cos 30 = 7.52 m/s
```

(**2** on the next page)

2. On a frictionless slope that makes an angle $\theta = 35°$ with the horizontal is a block of mass m, which is connected to another identical block of mass M with a massless string through a massless and frictionless pulley as illustrated below.



Let us choose the positive direction along the string as illustrated here.

(a) Suppose $m = M$. What is the magnitude of the acceleration of the blocks?[5]

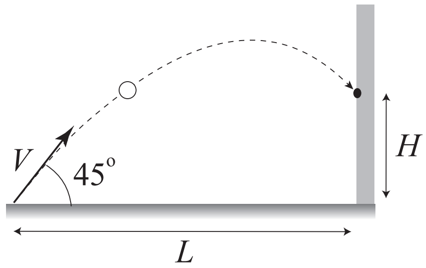
You must be able to draw the free-body diagrams for m and M.
Let us write down the equations of motion:
m: ma = T – mg sin \theta (upward along the slope is positive)
M: Ma = Mg – T (downward is positive)
Therefore,
(M + m)a = (M – m\sin 35)g,
or
a/g = (1- sin 35)/2 = 0.213.

(b) Suppose M is much larger than m (say, $M = 10^4 m$). What is the acceleration of the block of mass m? [5]

This is just a free fall of M, but m must follow it, so g.
Or you can take M/m -> infinity limit, to get a/g -> 1.

Name: ____________ **Section:** ________ **Score:** ________/20

1. We wish to aim at the target on the wall that is $L = 15$ m away at a height of $H = L/2 = 7.5$ m. You throw a ball with an initialvelocity of $\boldsymbol{V} = (V_x, V_y)$.



What is the initial speed $V = |\boldsymbol{V}|$? Let us solve this in two parts.

(a) In terms of the x-component V_x of the initial velocity, it takes the ball $t = L/V_x$ to reach the wall. Using this time, write down H in terms of the x-component of the initial velocity V_x ($= V_y$), L and the acceleration due to gravity g. [5]

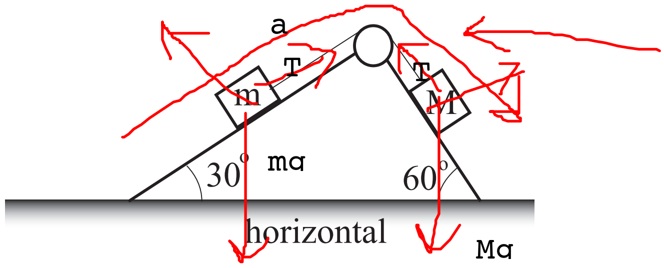
```
     H = 0 + V_y t -(1/2)gt^2,
so
     H = (V_y/V_x)L – gL^2/2V_x^2 = L – gL^2/V^2.
```

(b) Obtain V_x and then V. [5]

```
        H = L/2 = L – gL^2/V^2.
    That is,
        1/2 = gL/V^2.
    Therefore,
        V = \sqrt{2gL} = \sqrt{30g} = 17.15 m/s.
```

(**2** on the next page)

2. On frictionless slopes are two blocks of mass M and of mass m as illustrated below. They are connected by a massless cord through a massless and frictionless pulley.



Let us choose this direction along the string to be positive.

(a) Suppose $m = M$. What is the magnitude of the acceleration of the blocks? [5]

```
The equation of motion for
m: ma = T – mg sin 30, (upward along the slope is positive)
M: Ma = Mg\sin 60 – T.  (downward along the slope is positive).
Therefore,
(m + M)a = (M/2 – m\sqt{3}/2)g, or (sin 60 = \sqrt{3}/2)
a = (M\sqrt{3}/2 – M/2)/(M+m).  If M = m, we get
a = (\sqrt{3}/2–1/2)g/2 = 0.183g = 1.8 m/s^2
```

(b) Suppose M is much larger than m (say, $M = 10^4 m$). What is the magnitude of the acceleration of the block of mass m? [5]

```
This is just free sliding down of M along the slope, and m must
follow it, so
g sin 60  = 8.5 m/s^2 must be the answer.
```