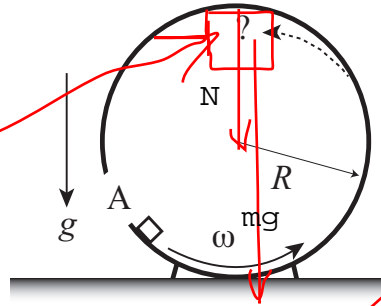




Do not mix up  $V$  and  $\omega$ ;  
 $V = R \omega$

Name: \_\_\_\_\_ Section: \_\_\_\_\_ Score: \_\_\_\_\_/20

1. A vertical hoop of radius  $R$  is fixed to the ground. Along its frictionless inside surface slides a block of mass  $m$ . It has a constant angular speed  $\omega$ .



both with the + signs., because they point to the center.

(a) Suppose  $R = 0.4$  m. What is the minimum (constant) angular speed  $\omega$  such that the block can reach the top of the hoop without falling off? [5]

You must be able to draw the free-body diagram when the block is at the top. The gravitational force is downward. There must be a force from the hoop  $N$  that must be downward (if any).

The motion is circular, so we must write the equation for the centripetal acceleration: its positive direction is always toward the center:



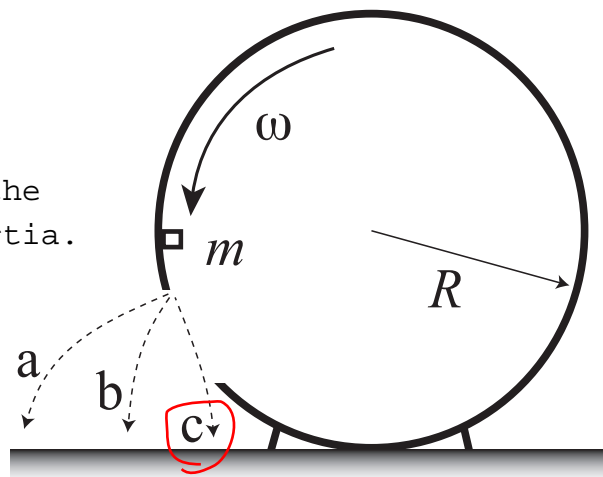
$$mR\omega^2 = mg + N \quad \text{this implies} \quad N = m(R\omega^2 - g).$$

$N > 0$  is required (you must 'feel' the wall!) to go around, so

$$\omega^2 > g/R = 24.5, \text{ so } \omega = 4.949 \text{ rad/s.}$$


(b) There is a gap at A, so the block falls off from the hoop on its way back to the bottom. Choose the (qualitatively) correct trajectory of the block after falling off the hoop. [5]

basically the law of inertia.



(2 on the next page)

2. DVD players read disks at a constant rate (linear speed) and thus the disk's rotational speed varies as it reads from the inner toward the outer edge of the disk.

 (a) To start a movie, 1500 rpm is needed to play the innermost track. What is the (minimum) angular acceleration (in rad/s) required to reach this rotational speed within 11 complete rotations? [5]

$$\omega^2 = \omega_0^2 + 2 a \Delta \theta \text{ is usable,}$$

since  $\omega$ ,  $\omega_0 (=0)$  and  $\Delta \theta = 11 \times 2\pi$  are given,

$$\omega = 1500 \times 2\pi/60 = 50\pi \text{ rad/s.}$$

Therefore,

$$(50\pi)^2 = 0 + 2a \text{ times } 22\pi.$$

That is,

$$a = (50\pi)^2/44\pi = 56.8\pi = 178.5 \text{ rad/s}^2$$

It is wise not to use the numerical value of  $\pi$  until the very last step.

(b) The radius of the innermost track is 2.3 cm. What is the *linear* speed required to play the outermost edge (its radius is 5.8 cm)? [5]

DVD requires the SAME linear speed everywhere on the disk, so we have only to calculate that around the innermost track:

$$\omega = 50 \pi \text{ as calculated above.}$$

$$R = 0.023 \text{ m.}$$

Hence,

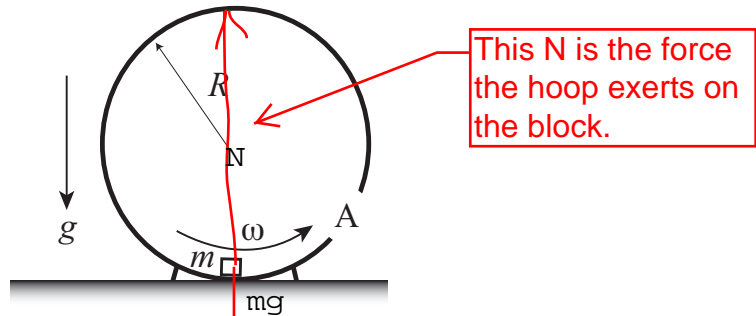
$$V = R \omega = 3.61 \text{ m/s.}$$

Do not mix up  $V$  and  $\omega$ ;  
 $V = R \omega$



Name: \_\_\_\_\_ Section: \_\_\_\_\_ Score: \_\_\_\_\_/20

1. A vertical hoop of radius  $R$  is fixed to the ground. Along its frictionless inside surface slides a block of mass  $m = 0.2$  kg. It has a constant angular speed  $\omega = 2.1$  rad/s.



(a) Suppose  $R = 1.2$  m. What is the force (its direction and magnitude) that the block exerts on the hoop when the block is just passing the bottom? [5]

Draw the free-body diagram: the gravitational force  $Mg$  (downward) and the normal force  $N$  from the hoop (upward).

Let us write down the centripetal acceleration. It is always toward the center of rotation. In this case 'upward' is positive.

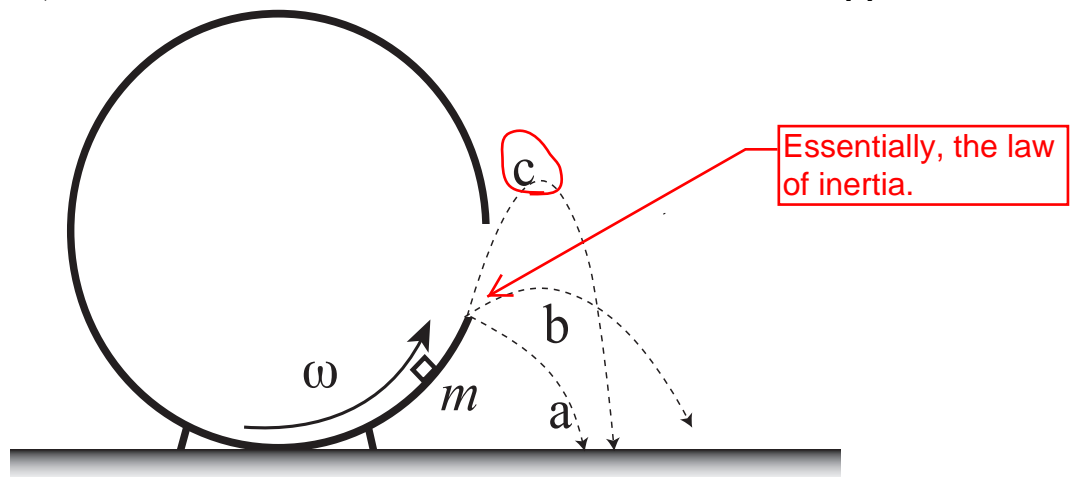
$$mR\omega^2 = N - mg \quad (\text{Pay attention to the signs}).$$

Therefore,

$$N = m(R\omega^2 + g) = 0.2 (1.2 \times 2.1^2 + 9.8) = 0.2 \times 15.092 = 3.02 \text{ N}.$$

The third law tells us the direction of the force the block exerts to the hoop: downward.

(b) There is a gap at A, so the block falls off from the hoop on its way to the top. Choose the (qualitatively) correct trajectory of the block after falling off the hoop. [5]



(2 on the next page)

2. CD players read disks at constant rate (linear speed) and thus the disk's rotational speed varies as it reads from the inner toward the outer edge of the disk.

(a) To start a song, 500 rpm is needed to play the innermost track. What is the (minimum) angular acceleration (in rad/s) required to reach this rotational speed within 8 complete rotations from the stationary state? [5]

x = times

Since,

$$\omega = 500 \times 2\pi/60 = 16.66\pi \text{ rad/s,}$$

$$\omega_0 = 0,$$

$$\Delta \theta = 8 \times 2\pi = 16\pi \text{ rad}$$

are given, we can use

$$\omega^2 = \omega_0^2 + 2 a \Delta \theta.$$

Therefore,

$$(16.66\pi)^2 = a \times 32 \pi,$$

or

$$a = 8.68 \pi = 27.3 \text{ rad/s}^2$$



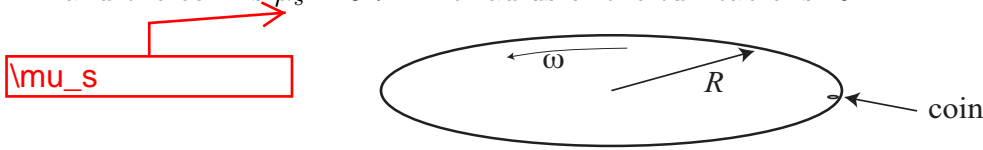
(b) The radius of the innermost track is 2.3 cm and that of the outermost track is 5.8 cm. What is the required rotational speed (in rpm) to play the outermost track? [5]

The linear speed  $R\omega$  must be identical. The rotational speed in rpm and  $\omega$  in rad/s are PROPORTIONAL. Therefore, (you need not worry about the units as long as the same units are used) the required rotation  $X$  in rpm must satisfy  $2.3 \times 500 = 5.8 \times X$ , so  $X = 2.3 \times 500/5.8 = 198.3$  (rpm).

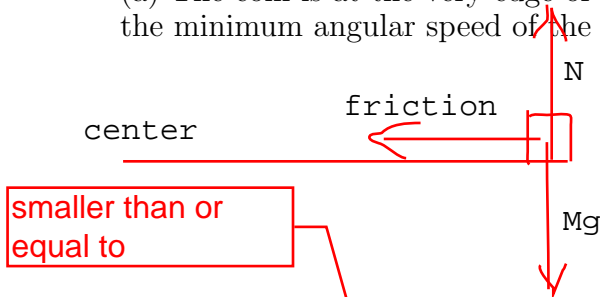
Do not mix up  $V$  and  $\omega$ ;  
 $V = R \omega$

Name: \_\_\_\_\_ Section: \_\_\_\_\_ Score: \_\_\_\_\_/20

1. On a horizontal turntable is a coin. The coefficient of static friction between the table and the coin is  $\mu_s = 0.7$ . The radius of the turntable is  $R = 1.2$  m.



(a) The coin is at the very edge of the table (you may ignore the size of the coin). What is the minimum angular speed of the turntable for the coin to fall off the table? [5]



Horizontal view

Friction is needed to supply the centripetal acceleration required for the coin to go around.

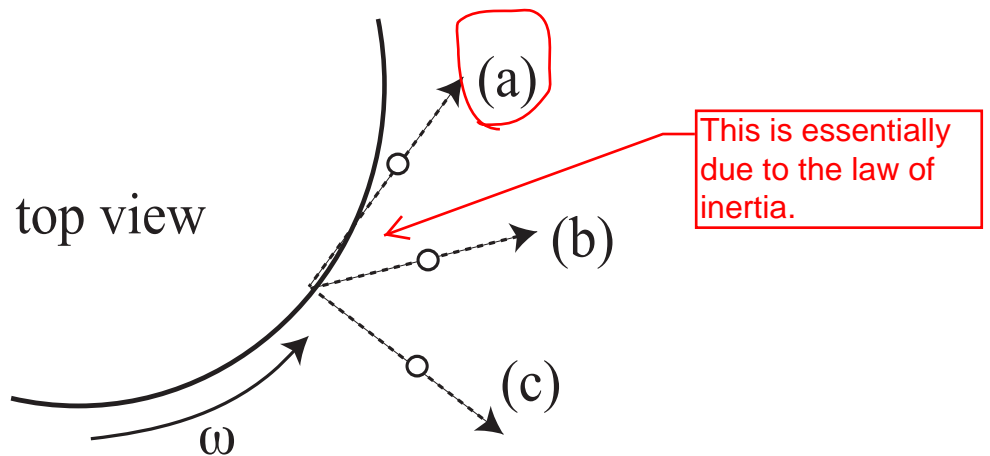
$$m R \omega^2 = f \text{ (the sign is +; to the center!)}$$

We know  $f \leq \mu_s N = \mu_s mg$ . Therefore, the 'critical speed' is given by

$$m R \omega^2 = \mu_s mg \text{ or } \omega^2 = \mu_s g / R = 5.72.$$

That is,  $\omega = 2.39$  rad/s.

(b) When the coin falls off the turntable to the frictionless floor (at the same height) due to a larger angular speed than that in (a), what is its qualitatively correct trajectory? Choose the right answer from below. [5]



(2 on the next page)

2. A centrifuge spins a sample at a distance 0.1 m from its axle at a rotational speed of 1800 rpm.

(a) We wish to apply the same centripetal acceleration to a sample held at a distance 0.07 m from the axle. What is the required rotational speed in rpm? [5]

The centripetal acceleration is  $R\omega^2$ . We wish to keep this value. Since  $\omega$  in rad/s and rotational speed in rpm are proportional,  $0.1 \times 1800^2 = 0.07 X^2$ , where  $X$  is the required rotational speed in rpm for the 0.07 m case. Therefore,

$$X = 1800 (0.1/0.07)^{1/2} = 1800 \times 1.195 = 2151 \text{ rpm.}$$

(b) We wish to accelerate the rotational speed from 1800 rpm to 3000 rpm within 230 rotations. What is the minimum angular acceleration do we have to apply to the centrifuge? [5]

Since

$$\omega = 3000 \times 2\pi/60 = 100\pi$$

$$\omega_0 = 1800 \times 2\pi/60 = 60\pi$$

and

$$\Delta \theta = 230 \times 2\pi$$

are given, we can use

$$\omega^2 = \omega_0^2 + 2 a \Delta \theta.$$

This  $a$  is the minimal angular acceleration:

$$(100\pi)^2 - (60\pi)^2 = 920 \pi a$$

That is,

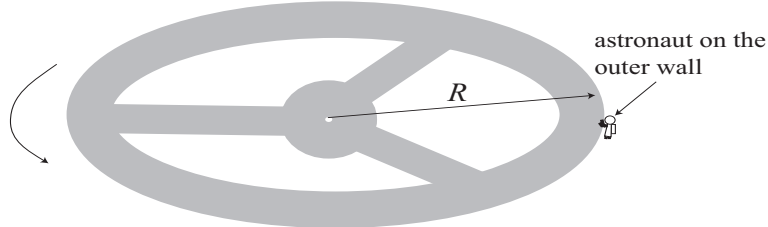
$$a = 6400\pi/920 = 6.96 \pi = 21.85 \text{ rad/s}^2.$$

It is wise to use the symbol  $\pi$  until the very last stage of the calculation

Do not mix up  $V$  and  $\omega$ ;  
 $V = R \omega$

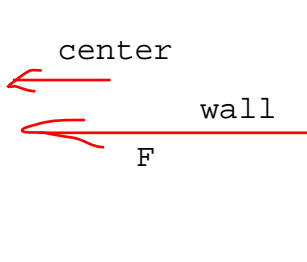
Name: \_\_\_\_\_ Section: \_\_\_\_\_ Score: \_\_\_\_\_/20

1. A donut shaped space station is rotating around its rotational symmetry axis as illustrated below. Its outermost radius is  $R = 120$  m, and it is rotating at an angular speed of  $0.35$  rad/s. An astronaut is clinging to the outer surface of the space ship.



(a) The mass of the astronaut is  $M = 120$  kg. What force (its direction and magnitude) must she exert on the wall to stay on the wall of the station? [5]

The free-body diagram is very simple; There is NO  $g$  in space! There is only one force  $F$  pulling the astronaut to the wall, which gives the centripetal acceleration.



$$MR\omega^2 = F$$

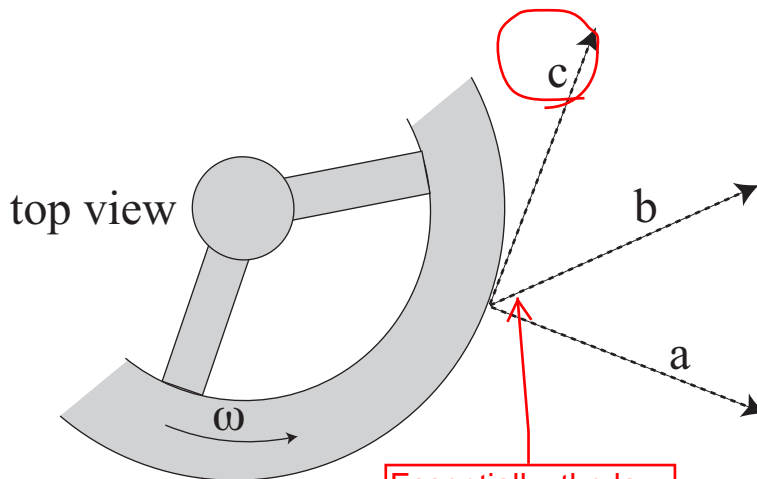
(Do not confuse  $\omega$  and the rotational speed  $R\omega$ ). This gives the magnitude of the force:

$$F = 120 \times 120 \times 0.35^2 = 1764 \text{ N.}$$

The  $F$  in the figure is the force the wall is exerting on the astronaut. Therefore, the third law (action-reaction) tells us that the force due to the astronaut is outward (the thick arrow)

side view

(b) She releases her grips and off she is gone to the depth of the universe. What is her trajectory just after she detaches from the wall? Choose the correct one from below. [5]



Essentially, the law of inertia

(2 on the next page)

2. An ancient phonogram used a vinyl disk that rotates at a constant rotational speed of  $100/3$  rpm.

(a) The playing speed (the linear relative speed of the needle and the disk)  $v_i$  at the beginning of the music (the outermost edge of radius = 14.5 cm) and that  $v_f$  at its ending part (the innermost grooves of radius = 5 cm) are different. What is the ratio  $v_i/v_f$ ? [5]

In contrast to CD or DVD that keeps the constant linear playing speed, the ancient disk keeps the rotational speed, so the linear playing speed changes from place to place.

$$v = R\omega \text{ and } \omega \text{ is constant.}$$

Therefore,

$$v_i/v_f = R_i/R_f = R(\text{outer})/R(\text{inner}) = 14.5/5 = 2.9.$$

Do not perform any unnecessary calculations.

(b) To stop the rotation within 5 rotations, what angular acceleration is required? [5]

The following quantities are given:

$$\omega = 0$$

$$\omega_0 = (100/3)2\pi/60 = 10\pi/9 \text{ rad/s}$$

and

$$\Delta \theta = 5 \times 2\pi = 10 \pi.$$

Therefore, we can use

$$\omega^2 = \omega_0^2 + 2 a \Delta \theta.$$

$$0 = (10\pi/9)^2 + 2a \times 10\pi.$$

Thus,

$$a = - [(10/9)^2/20]\pi = -0.0617 \pi = -0.194 \text{ rad/s}^2.$$

It is wise not to use  $\pi = 3.1415..$   
till the very last moment.