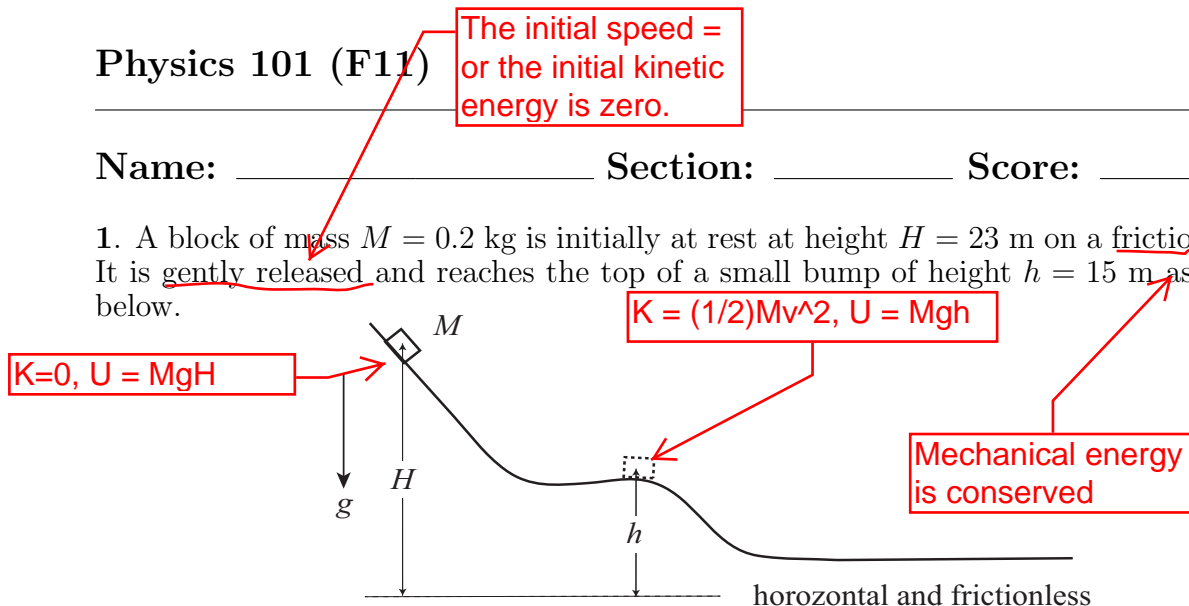


Name: _____ Section: _____ Score: _____/20

1. A block of mass $M = 0.2$ kg is initially at rest at height $H = 23$ m on a frictionless slope. It is gently released and reaches the top of a small bump of height $h = 15$ m as illustrated below.



(a) What is the speed of the block at the top of the bump? [5]

Mechanical energy is conserved:

Initial $K_i = 0$, $U_i = MgH$

Final $K_f = (1/2)Mv^2$, $U_f = Mgh$.

Therefore,

$$MgH = (1/2)Mv^2 + Mgh \quad \text{or} \quad (H-h)Mg = (1/2)Mv^2$$

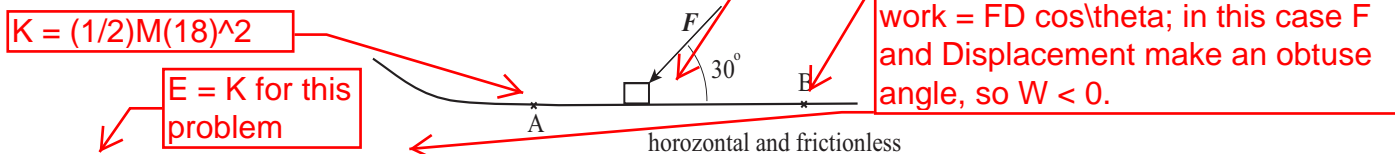
That is,

$$v = \sqrt{2(H - h)g} = 4\sqrt{g} = 12.52 \text{ m/s}$$

$W = -F D \cos 30$

$K = 0$

(b) The speed of the block is $V = 18$ m/s when it reaches the bottom horizontal portion. Then, a constant external force F starts to be applied to the block at point A in the figure below, and it comes to a halt at B (before reversing its velocity). Suppose the magnitude of the force $F = |\mathbf{F}| = 32$ N. What is the distance between A and B? [5]



$$E_f - E_i = -F \cos(30)D = -16\sqrt{3}D \quad (\text{the work-energy theorem})$$

$$E_i = (1/2)(0.2)18^2 = 32.4 \text{ J}$$

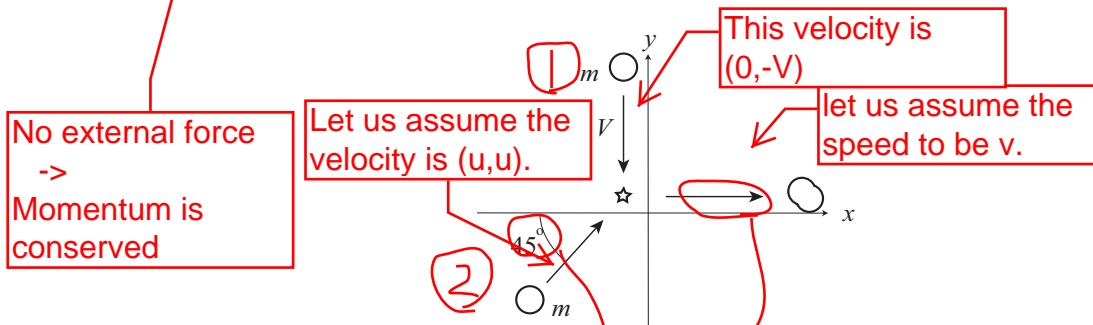
$$E_f = 0.$$

Therefore,

$$32.4 = 16\sqrt{3}D \quad \rightarrow \quad D = 1.169 \text{ m}$$

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2. Two pucks of the same mass M are moving as illustrated (top view) in the figure on a frictionless horizontal floor. The puck coming along the y -axis has a speed of V . They collide around the star mark, and stick together.



(a) After sticking together two pucks move along the x -axis. What is its speed in terms of V ? [5]

For (1) $V_1 = (0, -V)$

For (2) $V_2 = (u, u)$

The total momentum is conserved

$$\text{Initial } P_i = m(0, -V) + m(u, u) = m(u, u - V)$$

$$\text{Final } P_f = 2m(v, 0).$$

This means $2v = u$ and $u = V$. Thus, $v = V/2$.

OR

More intuitively,

Cancellation of y -components implies the x -component of (2) velocity is V . Therefore, the conservation of the x -component of the total momentum implies $2Mv = MV$, or $v = V/2$.

(b) What is the kinetic energy loss due to the collision? Assume $M = 1$ kg and $V = 1$ m/s. [5]

for (2)

for (1)

There is no simple trick.

$$\text{Initial: } K_i = (1/2)M(V^2 + V^2) + (1/2)MV^2 = (3/2)MV^2$$

$$\text{Final: } K_f = (1/2) 2M (V/2)^2 = MV^2/4.$$

Therefore,

$$K_f - K_i = -(5/4)MV^2 = -1.25 \text{ J, i.e, } 1.25 \text{ J lost.}$$

That the mechanical energy is not conserved in this collision can be seen immediately from the vanishing of the y -component of the velocity.

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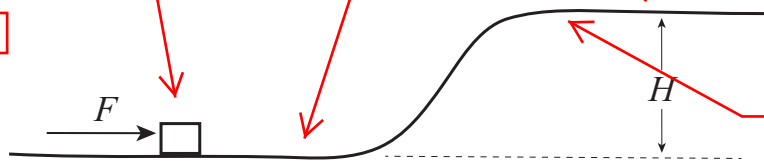
W done by the force, but in this problem $W = FD$ is inconvenient

$$K = P^2/2M$$

$$U = 0$$

1. To push up a block of mass $M = 1.2$ kg to a higher floor of height $H = 12$ m, a horizontal force of magnitude $F = 12$ N is applied to the block to the right for 2 s before it reaches the slope as illustrated. The block successfully overcomes the slope and reaches the higher floor. The floor and the slope are assumed to be frictionless.

$$K = U = 0$$



Mechanical energy is conserved

$$U = MgH, \text{ so}$$

$$K = P^2/2M - MgH$$

(a) What is the momentum and the kinetic energy of the block just before it reaches the slope? [5]

To obtain the momentum, we use Newton's second law (since there is a force!) $\Delta P / \Delta t = F$, so $P = 12 \times 2 = 24$ kg.m/s

$$K = P^2/2M = 24^2/2.4 = 240 \text{ J.}$$

$$\text{kg.m/s} = \text{Ns}$$

(b) What is the speed of the block after reaching the higher floor? [5]

After F is turned off, mechanical energy is conserved.

Before the slope $K_i = 240 \text{ J}, U_i = 0,$

After the slope $K_f = (1/2)MV^2, U_f = MgH = 1.2 \times 12 \times 9.8 = 141.1\text{J.}$

Therefore,

$$240 = (1/2) \times 1.2 V^2 + 141,$$

i.e.,

$$V^2 = 99/0.6 \text{ that is } V = 12.85 \text{ m/s}$$

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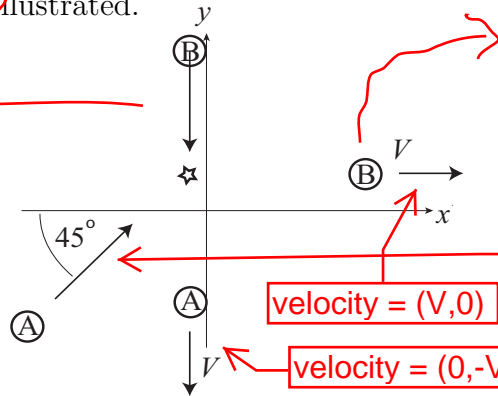
It should have been stated that the floor is horizontal and frictionless. (sorry)

2. Two pucks A and B with the same mass m collide around the star mark in the figure below. They have the same speed V after the collision; puck A leaves along the y -axis, and puck B along the x axis as illustrated.

$$V_B = (0, -u_B)$$

$$V_A = (u_A, u_A)$$

They are initially unknown.



Its x-component is V , so the initial y-component of the velocity of A is V . Thus, initial speed of B can be obtained from this information, but here let us solve the problem 'mechanically' in terms of vectors.

$$\text{velocity} = (V, 0)$$

$$\text{velocity} = (0, -V)$$

(a) What is the initial speed of puck B in terms of V ? [5]

The momentum is conserved:

$$\text{Initial: } P_i = m(u_A, u_A) + m(0, -u_B) = m(u_A, u_A - u_B)$$

$$\text{Final: } P_f = m(0, -V) + m(V, 0) = m(V, -V).$$

Momentum conservation implies $P_i = P_f$, so

$$(u_A, u_A - u_B) = (V, -V).$$

Or

$$u_A = V, \quad u_A - u_B = -V.$$

Therefore

$$u_B = 2V.$$

This should be obvious from this explanation.

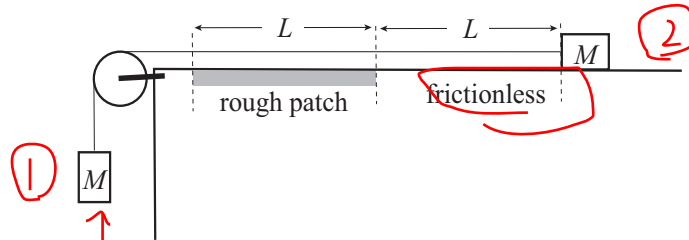
(b) Is the mechanical energy conserved? Justify your answer. [5]

No. The y -components already violate the energy conservation.

Name: _____ Section: _____ Score: _____/20

1. Two blocks of the same mass M are connected with a massless string through an ideal pulley. Initially, the system is stationary. Then, the hanging block is gently released. The table top is horizontal and frictionless except for the rough patch.

DO NOT forget that BOTH blocks move at the same speed!



(a) What is the speed v of the block on the table when it reaches the rightmost edge of the rough patch after running the distance L in terms of L and g , the acceleration due to gravity? [5]

Mechanical energy is conserved.

Initial: (Since (1) goes down by distance L , the potential energy origin should be chosen to be here. $K_i = 0$, $U_i = MgL$.)

Final: $K_f = (1/2)MV^2 + (1/2)MV^2$, $U_f = 0$.

Both M moves with the same speed V

Hence,

$$MgL = MV^2 \quad \text{that is, } V = \sqrt{gL}.$$

(b) While running inside the rough patch, the speed of the block is constant. What is the total work W (its magnitude) done by the friction force to the block in terms of L , M and g ? [5]

This implies that the kinetic energy stays constant. However, Block (1) still goes down, so its potential energy should have been converted to the kinetic energy. However, this does not happen, so $\Delta U = MgL$ must be dissipated by friction. Therefore, the work done to the block by friction is (negative of) $W = MgL$.

(2 on the next page)

This is a disguised (inelastic) collision problem.

2. A rifle bullet of mass 2.6 g flying horizontally at 950 m/s hits a block of mass 1.2 kg stationary on a frictionless floor.

(a) The bullet goes through the block, emerging with a speed of 230 m/s. What is the speed of the block after the collision? [5]

v

The horizontal component of momentum is conserved, because there is no external force in the horizontal direction.

Initial momentum: 0.0026×950

Final momentum: $0.0026 \times 230 + 1.2V$

$$0.0026 \times (950 - 230) = 1.2V \quad \text{so } V = 1.56 \text{ m/s}$$

(b) Is the mechanical energy conserved? Justify your answer. [5]

No. This is almost obvious, if you understand that the situation is almost the same as a bullet going through a wall, a heavier object than the bullet.

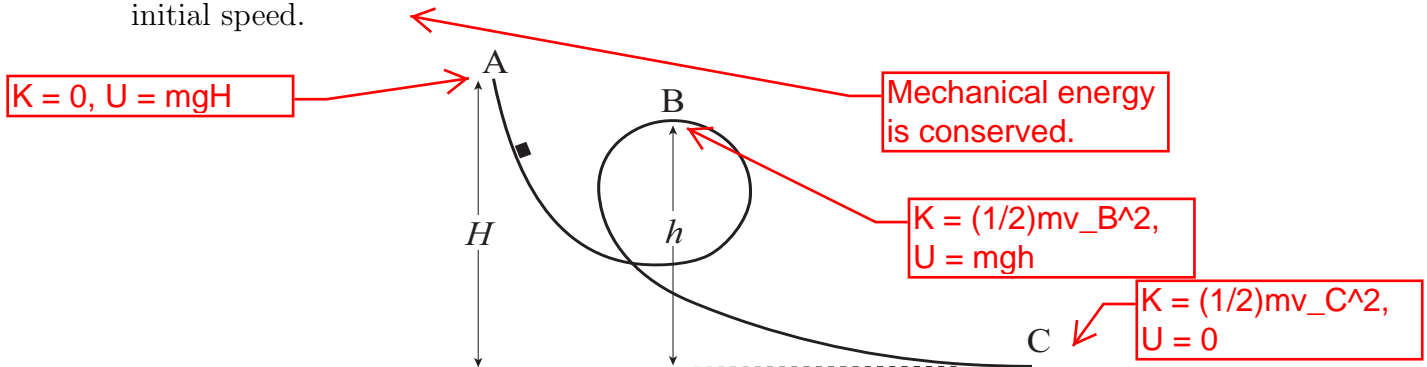
Or

Compare V^2 of the bullet and the block, the ratio of the kinetic energy is of order m/M , so crudely speaking (of order) $100(1-m/M)\%$ of energy is lost.

mass of bullet mass of block

Name: _____ Section: _____ Score: _____/20

1. Along a frictionless toy coaster track is a block of mass m sliding down from A with zero initial speed.



(a) The ratio of the speed at C to the speed at B is 3. What is the ratio of the heights h/H ? Assume that the block does not fall off the track. [5]

Conservation of mechanical energy

At A: $K_A = 0, U_A = mgH$

At B: $K_B = (1/2)mv_B^2, U_B = mgh$

At C: $K_C = (1/2)mv_C^2, U_C = 0$ (C is chosen to be the origin of the height)

Therefore,

$$v_B^2 = 2g(H - h),$$

$$v_C^2 = 2gH.$$

This implies

$$H/(H - h) = 9.$$

That is,

$$1 - h/H = 1/9 \quad \text{or} \quad h/H = 8/9.$$

(b) The speed at B is actually 1.1 m/s. What is $H - h$? [5]

$$H - h = v_B^2/2g = 1.1^2/2g = 0.0617 \text{ m.}$$

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Still the total momentum must be 0

Initial total momentum = 0

2. There are two space ships with the same mass M which are both initially stationary (relative to distant stars). Spaceship A has a cargo of mass m (i.e., its total initial mass is $M + m$). This cargo is transferred to Spaceship B.

Be careful.

(a) The cargo is pushed out from Spaceship A with a speed of v relative to Spaceship A. What is the speed of Spaceship A relative to distant stars in terms of v , m and M ? [5]

Let V_A be the velocity of the spaceship A relative to distant stars
Let U be the velocity of the cargo relative to distant stars.

Momentum is conserved

Initial: $P_i = 0$

Final: $P_f = M V_A + mU$, that is, $MV_A + mU = 0$.

Since v is the relative speed of the cargo, $U - V_A = v$.

Therefore,

$$MV_A + m(v + V_A) = 0.$$

That is,

$V_A = -mv/(M + m)$. (The sign may be ignored to obtain the magnitude.)

(b) The cargo is received by Spaceship B. What is the ratio of the speeds of the spaceships v_A/v_B , where v_A (respectively, v_B) is the speed of Spaceship A (respectively, B) after the transfer? [5]

both must be relative to distant stars

For these three object the total momentum is zero, because initially it was zero.

Therefore

$$Mv_A + (M+m)v_B = 0$$

That is,

$$v_A/v_B = -(1 + m/M).$$

Thus, the speed ratio is $1 + m/M$.

This implies that now B and the cargo move together with the same velocity v_B .

Notice that this problem has nothing to do with (a), totally decoupled.