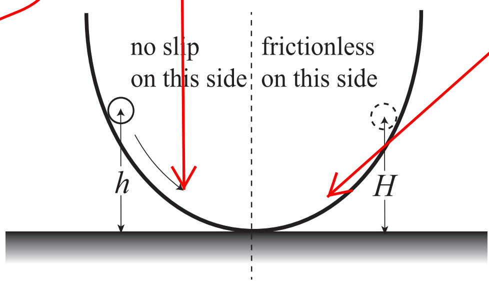
All the initial potential energy is converted into the kinetic energy at the bottom.

Name: ________________ **Section:** __________ **Score:** __________/20

1. A uniform disk (which will be replaced by a uniform hoop in (b)) starts to slide down the slope on the left side without slip from a stationary condition at height h, and then climbs up the right side slope which is frictionless.

Conservation of mechanical energy



However, the rotational kinetic energy cannot be utilized to climb up this frictionless slope.

(a) The height of the highest point reached by the disk on the right side slope is H. What is the ratio H/h? [Hint: You may assume the mass M and the radius R of the disk, although they are not needed.] [5]

```
The initial potential energy U = Mgh is totally converted to the
kinetic energy at the bottom: K = K_T + K_R = Mgh (conservation!).
Then, the disk starts to climb up the right slope.  This is
frictionless, so the rotation (or the angular speed) cannot change!
That is, only K_T is converted into the final potential energy, so
MgH = K_T (at the bottom).
     As you see, this is, essentially the K_T/K_R ratio problem.
MgH/Mgh = K_T/(K_T + K_R) = (1/2)MV^2/[(1/2)MV^2 + (1/4)MV^2] = 2/3.
Here, V is the translational speed at the bottom, so
     K_T = (1/2)MV^2    K_R = (1/2)[(1/2)MR^2]\omega^2 = (1/4)MV^2
```

at the bottom

(b) If the same experiment is repeated with the disk replaced by a uniform hoop of the same radius, is the ratio H/h larger (i.e., does the hoop climb up higher than the disk)? You must justify your answer. [5]

```
Now, the disk is replaced by a hoop which has a larger I.  This
implies that the hoop has `more share of K_R,' so less kinetic energy
can be used to climb up the right slope. Therefore, H/h is smaller.
If you wish
     K_T = (1/2)MV^2
     K_R = (1/2)I\omega^2 = (1/2)MR^2\omega^2 = (1/2)MV^2.
Therefore,
     MgH/Mgh = K_T/(K_T + K_R) = (1/2)MV^2/[(1/2)MV^2 + (1/2)MV^2]
             = 1/2.  Or, H = h/2.
```

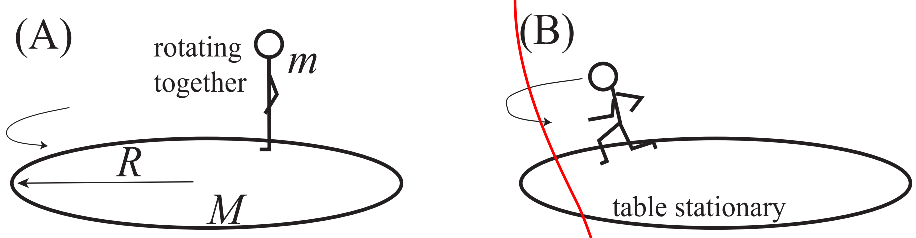
If M and R are the same. (Actually, only the shape matters or I/MR^2 matters.)

(2 on the next page)

No external torque -> conservation of angular .
You cannot say anything about the total kinetic energy from the no torque condition.

2. A uniform horizontal turntable of radius R and mass M can rotate around a vertical axle without friction. Initially, a person of mass $m = M/2$ is standing still on the turntable at its edge (R from the axle). The standing person and the turntable rotate together with the same angular speed ω (A in the figure).

Angular momentum is conserved.



(a) The person starts to run along the edge of the turntable and the turntable comes to a complete halt (situation B). What is the angular speed Ω of the person around the axle? [5]

```
Initial angular momentum
     L = (I + mR^2)\omega = [(1/2)MR^2 +(1/2)MR^2]\omega
       = MR^2\omega.
Final angular momentum
     L = mR^2\Omega = (1/2)MR^2\Omega
Therefore,
     \Omega = 2\omega.
```

The table and the person is rotating together around the same axle, so the total moment of inertia is the sum around the same rotation center: I + mR^2

(b) Does the total kinetic energy increase or decrease from A to B? You must justify your answer. [5]

```
K = L^2/2I.
We know L is conserved, so K changes only due to the change of I.
If I is reduced, then K is increased.  This is exactly what
happens.
     I before = MR^2
     I after = MR^2/2.
Therefore, K after = 2 K before.
```

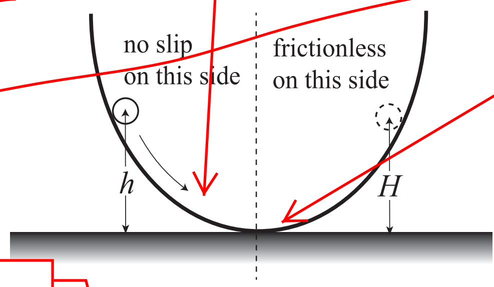
All the initial potential energy is converted into the kinetic energy K at the bottom. Without slip K consists of translational and rotational parts.

Physics 101 (F11) **Q8B**

Name: ______________ **Section:** ________ **Score:** ________/20

1. A uniform solid ball (which will be replaced with a hollow sphere in (b)) starts to slide down the slope on the left side from a stationary condition at height h without slip, and then climbs up the right side frictionless slope.

Mechanical energy is conserved,



However, the rotational kinetic energy cannot be utilized to climb up this frictionless slope.

at the bottom

(a) The height of the highest point reached by the solid ball on the right side slope is H. What is the ratio H/h? [Hint: You may assume the mass M and the radius R of the ball, although these are not needed.] [5]

```
The initial potential energy U = Mgh is totally converted to the
kinetic energy K = K_T + K_R = Mgh (conservation!).
Then, the ball starts to climb up the right slope.  This is
frictionless, so the rotation (or the angular speed) cannot change!
That is, only K_T is converted into the final potential energy, so
MgH = K_T.
As you see, this is, essentially the K_T/K_R ratio problem.
MgH/Mgh = K_T/(K_T + K_R) = (1/2)MV^2/[(1/2)MV^2 + (1/5)MV^2] = 5/7.
Here, V is the translational speed at the bottom, so
     K_T = (1/2)MV^2   K_R = (1/2)[(2/5)MR^2]\omega^2 = (1/5)MV^2
```

No slip means V = R \omega

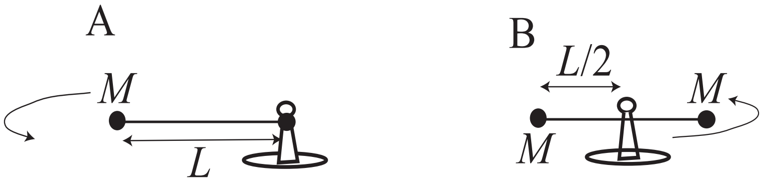
(b) If the same experiment is repeated with the solid ball replaced by a uniform hollow sphere of the same radius, is the ratio H/h larger (i.e, does the sphere climb up higher than the ball)? You must justify your answer. [Hint: You may assume the ball and the sphere have the same mass, although this information is not needed.] [5]

```
Now, the ball is replaced by a sphere which has a larger I.  This
implies that the sphere has `more share of K_R,' so less kinetic energy
can be used to climb up the right slope. Therefore, H/h is smaller.
     K_T = (1/2)MV^2
     K_R = (1/2)I\omega^2 = (1/2)[(2/3)MR^2]\omega^2 = (1/3)MV^2.
Therefore,
     MgH/Mgh = K_T/(K_T + K_R) = (1/2)MV^2/[(1/2)MV^2 + (1/3)MV^2]
             = 3/5.  or H = 3h/5.
```

(**2** on the next page)

2. On a horizontal turntable, which can rotate around its vertical axle freely but whose moment of inertia you may ignore, is a person holding a massless stick of length L with a tiny ball of mass M at each end. Initially, she holds the stick horizontally by one end as in A, and is rotating with the stick at an angular speed ω.

angular momentum around the turntable center.



Angular momentum is conserved.

(a) She pulls in the stick and holds it horizontally by the mid point as in (B). Her new angular speed is Ω. What is Ω/ω? You may ignore her moment of inertia (for simplicity). [5]

```
Initial: AngMom = ML^2\omega, because I = ML^2
Final: I = M(L/2)^2 x 2 = ML^2/2.  AngMom = (ML^2/2)\Omega.
Therefore, ML^2\omega = (ML^2/2)\Omega  or \Omega/\omega = 2.
```

In this problem L is used for the length of the stick; it is NOT the angular momentum.

Never be fooled by the symbols.

It should be intuitively clear that \Omega is larger than \omega.

(b) What is the work W done by the person in terms of M, L and ω? [5]

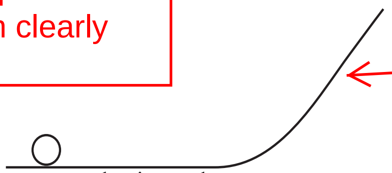
```
The rotational kinetic energy reads
     K = (AngMom)^2/2I,
where AngMom is conserved.
If I is reduced, K is increased.  This is just what happens.

Initial kinetic energy = (ML^2\omega)^2/2ML^2 = (1/2)ML^2\omega^2
Final kinetic energy = (ML^2\omega)^2/ML^2 = ML^2\omega^2
Therefore, the work-energy theorem W = \Delta K implies
     W = ML^2\omega^2 - (1/2)ML^2\omega^2 = (1/2)ML^2\omega^2.
```

Name: ____________________ **Section:** __________ **Score:** __________/20

1. Initially, a uniform disk has a speed of v, and then climbs up the slope, reaching the maximal height h.

without slip should have been clearly stated.



(a) Then, oil is applied to the surface, and the floor and the slope become frictionless. The same experiment is repeated with the same initial speed, and the disk climbs up to the height H. What is H/h? [5]

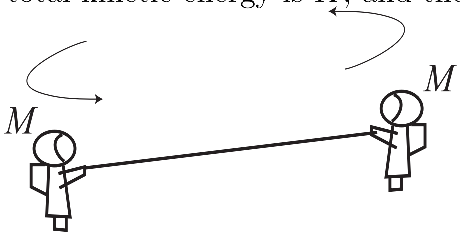
```
w/o Oil: K = K_T + K_R = (1/2)mv^2 + (1/2)(mR^2/2)(v/R)^2 = (3/4)mv^2.
     This total kinetic energy is converted to U = mgh.
with Oil: now rotation does not matter.  K = K_T = (1/2)mv^2.
     This total kinetic energy is converted to U = mgH.
Therefore,
     mgh = (3/4)mv^2
     mgH = (1/2)mv^2.
Thus,
     H/h = 2/3.
```

(b) Now, the disk is replaced with a uniform hoop of the same size and the experiments with and without oil in (a) are repeated. Does the ratio H/h increase? You must justify your answer. [5]

```
If the disk is replaced by the hoop (whose moment of inertia is larger
than that of the disk), so the ratio must decrease.
     Detailed calculation:
w/o Oil: K = K_T + K_R = (1/2)mv^2 + (1/2)(mR^2)(v/R)^2 = mv^2.
     This total kinetic energy is converted to U = mgh.
with Oil: now rotation does not matter.  K = K_T = (1/2)mv^2.
     This total kinetic energy is converted to U = mgH.
Therefore,
     mgh = mv^2.
     mgH = (1/2)mv^2.
Thus,
     H/h = 1/2.
```

(**2** on the next page)

2. Two astronauts with the same mass M are pulling each other and rotating around their center of mass. Initially, the total kinetic energy is K, and the angular speed is ω.



(a) What is the distance between the astronauts in terms of K, M and ω? [5]

```
Let D be the distance.
The moment of inertia I of the system is I = M(D/2)^2 x 2 = MD^2/2.
The rotational kinetic energy K
      K = (1/2)I\omega^2 = M(D\omega)^2/4.
Therefore,
     4K/M = (D\omega)^2
or
     D = (2/\omega)(K/M)^{1/2}.
```

(b) The astronauts pull each other and the mutual distance is halved. Let the new kinetic energy be K'. What is the ratio K'/K? [5]

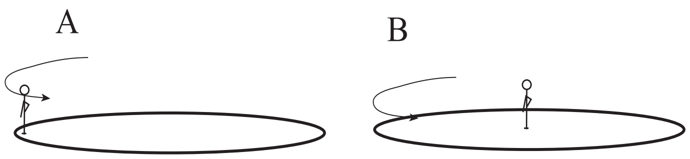
```
The angular momentum L is conserved.
     K = L^2/2I,
where
     I before = M(D/2)^2 x 2 = MD^2/2.
     I after = M(D/4)^2 x 2 = MD^2/8.
Therefore,
     K after/K before = I before/I after = 4.
```

Physics 101 (F11) **Q8D**

it should have been stated that the system is without friction around the axle

Name: **…n:** ______ **Score:** ______ **/20**

1. A horizontal turntable of moment of inertia I and radius R can rotate around its central vertical axle. Initially, the turntable is rotating at a constant angular speed ω together with a person of mass m standing at the edge of the table stationary relative to the table (Situation A). Throughout this problem you may assume that the person is a point mass (= mass of insignificant size.)



(a) The angular speed of the turntable is Ω after the person walks to the center and stands just at the position of the axle (i.e. Situation B). What is Ω/ω in terms of I, m and R? [5]

```
The angular momentum is conserved.
Before: moment of inertia = I + mR^2, L = (I + mR^2)\omega
After: moment of inertia = I, L = I\Omega.
Therefore,
     \Omega/\omega = 1 + mR^2/I.
That is, the table rotates faster than before.
```

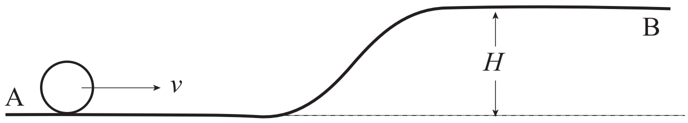
(b) Does the person do a positive work between Situation A and Situation B? You must justify your answer. [5]

```
Yes, since the moment of inertia is reduced.

We know
     K = L^2/2I
L is conserved, and I is reduced, so K increases.
```

(**2** on the next page)

2. There are two horizontal floors A and B, the latter being higher by H than the former.



Energy is conserved.

(a) A disk is given an initial translational speed of v. The disk rolls without slip all the way up to floor B, but comes to a halt. What is H in terms of v and g, the acceleration due to gravity? [5]

The final K = 0

```
K = (1/2)mv^2 + (1/2)I(V/R)^2 = (3/4)mv^2
This is exactly converted to the potential energy U = mgH.
Therefore,
     H = 3v^2/4g.
```

(b) Now, oil is applied all over, and the floors and slope are frictionless. Then, the experiment is repeated with the same disk with the same initial translational speed v. Can the disk reach floor B? You must justify your answer. [5]

```
No.  Only the translational kinetic energy can be used to go up
the slope.
If we repeat the calculation above:
K = (1/2)mv^2  (only translational; there may be K_R but it cannot
change, so we may ignore it.)
This is exactly converted to the potential energy U = mgh at the highest
point of height h.
Therefore,
     h = v^2/2g = 2H/3.
```