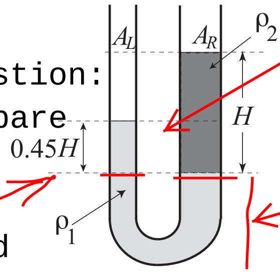
Physics 101 (F11) Don't be fooled by red herrings. **Q9A**

You must immediately guess this is > 1.

Name: ____________ **Section:** ________ **Score:**

1. A vertical U-tube described below contains two distinct liquids with densities ρ_1 and ρ_2, respectively, which do not mix. The cross sectional area of the left column is $A_L = A$ and that of the right $A_R = 3A$. The ends of the tube are open to the air. The stationary state of these liquids in the tube is illustrated in the figure below. What is the ratio ρ_1/ρ_2? [5]

This is a typical Pascal's principle (with gravity) question: Remember: you can easily compare pressures
(1) at the same height
(2) in the same kind of fluid



Here, the same heights do not guarantee the same pressure, because fluids are distinct.

Here, the same heights implies the same pressure.

We need one more ingredient: pressure-depth relation P(d) = P(0) + g\rho d.
Pressures:

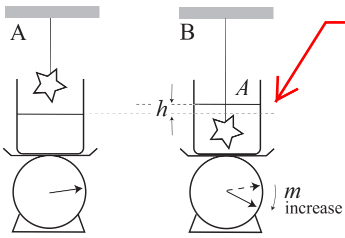
Left at the red line: P = P_A + \rho_1 0.45H g,

Right at the red line: P = P_A + \rho_2 H g.

Therefore, 0.45 \rho_1 = \rho_2. So the ratio is 2.22.

Here, the atmospheric pressure are common, so you may ignore it.

2. When an object of mass M hanging from above is immersed (without touching the bottom; see the illustration below) into a liquid of density ρ in a container of cross sectional area A below, the surface of the liquid rises by h (See the change from A to B in the figure). What is the change of the reading of the scale on which the container sits? You may answer this as the mass change m or the weight change mg in terms of h, A, ρ, etc., where g is the acceleration due to gravity. [5]

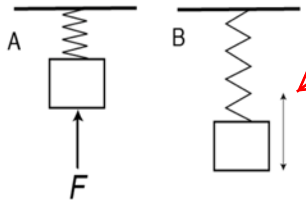


This tells you that the volume of the `star' is hA.

Mr Archimedes tells us that the buoyancy force acting on the `star' is \rho hA g (upward). This buoyancy force comes from the liquid; the action-reaction principle tells us that the liquid is pushed down by the same force. This must eventually push the scale. So m = \rho h A or mg = \rho hA g.

(**3** and **4** on the next page)

3. A block of mass M is suspended from the ceiling by a spring with spring constant k. Initially, the block is applied a force $\boldsymbol{F}$ from below as illustrated in A. Then, the force is turned off, and the block starts to oscillate as in B. What is the maximum acceleration a_M of the block in terms of $F = |\boldsymbol{F}|$, M and k? (You need not use all the quantities.) [5]



For oscillations around the equilibrium position you can totally forget about gravity.

```
You can immediately guess: F/M.  In more detail,
When F is removed, then the spring pushes the block with the force whose
magnitude is exactly F. Hence, F/M must be the acceleration.

Illustration `A' is actually the situation with the maximum displacement
(x = +/-amplitude), so the acceleration must be maximum.
```

You must immediately guess that this is < 1.

doesn't matter

4. A pendulum made from a string and a small weight of mass m attached to its end has a frequency f on the earth. Its frequency is f' on Europa whose acceleration due to gravity on its surface is 13.4 % of the earth's. What is f'/f? ([5]

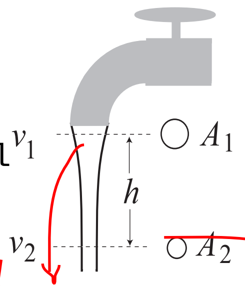
```
f = 2 \pi \omega = 2\pi \sqrt{g/L}.  Since we keep L,
f'/f = \sqrt{g'/g} = \sqrt{0.134} = 0.366.
```

Physics 101 (F11) Q9B

Name: ______________ **Section:** __________ **Score:** ________/20

1. A water column illustrated below is formed when a faucet is opened. A_1 and $A_2 = A_1/\sqrt{2}$ denote the cross sectional areas and v_1 and v_2 the speeds of water at the respective heights separated by distance h vertically in the illustration. Obtain v_1 in terms of h and g, the acceleration due to gravity. [5]

```
This must be a Bernoulli's
principle question.
Let us write down the
`conservation of mechanical
energy.'
```



Since the surrounding pressure is the ordinary atmospheric pressure, you can drop it from both sides of the equation.

Kinetic energy (density)

Potential energy (density)

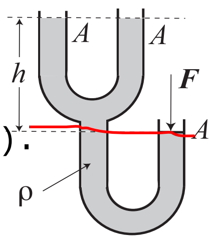
```
     (1/2)\rho v_1^2 + \rho gh + P_A = (1/2)\rho v_2^2 + P_A,
```

Stored energy (density) due to compression

```
which implies
     v_1^2 + 2gh = \v_2^2.
We have two unknowns: v_1 and v_2, so we need one more equation:
conservation of mass (continuity equation).  Water is incompressible,
so \rho does not change appreciably; v_1A_1 = v_2 A_2.  That is,
v_2 = \sqrt{2}v_1. Thus, we get from these two relations v_1^2 = 2gh.
Or, v_1 = \sqrt{2gh}.
```

2. A 'branched U-tube' illustrated below is filled with a liquid of density ρ. A force of magnitude F is pushing the piston in the right column vertically down and the liquid surface on the left columns is pushed up by h. The cross sectional area of the columns are all A. Now, the force is increased slightly, and the piston goes down by Δ. What is the change of the height of the liquid surfaces in the left collumns? (Or, obtain the change in h, if you can.) [5]

```
Obviously, \Delta/2.
This simply conservation
of mass (continuity equation).
h is increased by 3\Delta/2.
```

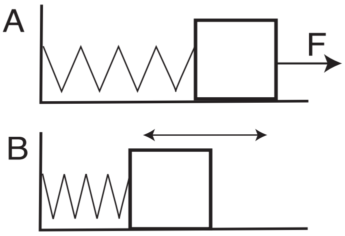


The atmospheric pressure is on both sides, so you need not write P_A.

```
We could ask for F.  Only
the head difference matters.
Don't be fooled by the
shapes and cross sections.

The pressure at the red line must be the same:
Left: P_A + \rho gh,
Right: P_A + F/A.

Therefore,
     F = \rho g h A.
```

(**3** and **4** on the next page)

3. A block of mass M is attached to a wall by a spring with spring constant k. Initially, a force of magnitude F pulls the mass to the right as in A. The block is at rest when the force is turned off. What is the maximum kinetic energy K_M of the block when it oscillates as in B in terms of F and k? Assume that the floor is horizontal and frictionless. [5]



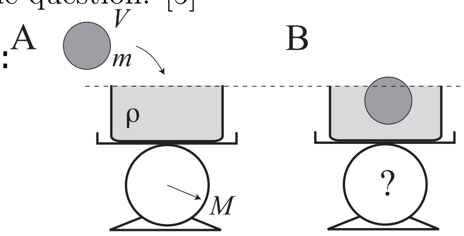

```
The displacement in Situation `A' is the amplitude A = F/k.
When x = A, the mechanical energy is totally potential, so
E = U = (1/2) kA^2 = F^2/2k. The maximum kinetic energy is E, so
     K_M = F^2/2k.
```

4. A pendulum is made from a string of length 1 m and a small weight of mass $m = 2$ kg attached to its end. The amplitude of the oscillation is $A = 0.1$ m. What is the maximum speed of the weight? You may use the small amplitude approximation. [5]

```
The maximum speed is A \omega.
     \omega = \sqrt{g/L} = \sqrt{9.8}.
Therefore,
     max speed = A\w = 0.1 \sqrt{9.8} = 0.31 m/s.
```

Name: ____________________ **Section:** __________ **Score:** __________/**20**

1. On a spring scale is a container that contains brimful of a fluid of density ρ as shown in A. The reading of the scale is M (kg). Now, a uniform ball of mass m and volume V is put in the container as shown in B. The ball floats and the liquid spills over. After removing the spilled fluid, what is the reading of the scale? You may use M, m, V and g (the acceleration due to gravity) to answer the question. [5]

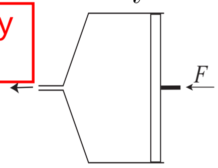



```
Probably, you can quess:
NO CHANGE: M.

In detail:

  The volume of the spilt fluid must be equal to the volume of the
  ball below the fluid surface.

  Why is the ball on the surface?  Because of the buoyancy = mg.
  This weight is added.
  Mr Archimedes tells us that the buoyancy = the weight of the
  displaced fluid = the weight of the spilt fluid = mg, lost.
  Hence, there is a total cancellation: NO change!
```

0.01 m^2

2. A water ejector has a piston of cross sectional area 100 cm^2 and a small nozzle of cross sectional area 1 cm^2. We wish the ejected water to have a speed of 10 m/s. What is the magnitude of the required force F? The density of water is $\rho = 1000\ \text{kg/m}^3$. [5]

The kinetic energy (density)



P_A, the atmospheric pressure acts on both sides, so you can ignore it from the start.

```
This is a Bernoulli
principle question.
Left: (1/2)\rho v^2 + P_A,
Right: P_A + F/A.
Thus,
     (1/2)\rho v^2 = F/A.
That is,
     F = (A/2)\rho v^2 = (0.01/2)1000(10^2) = 500 N.
```

The cross sections are so different that the continuity equation (or intuition) tells us that the piston moves only very slowly; you may ignore it.

(**3** and **4** on the next page)

3. A block of mass $M = 0.2$ kg on a horizontal and frictionless floor is attached to a wall by a spring with spring constant $k = 60$ N/m. When its displacement from the equilibrium position is 0.1 m, its speed is 3 m/s. What is the maximum speed of the block?[5]

```
The total mechanical energy E is conserved.
Max speed V satisfies
     E = (1/2)MV^2.

Generally,
     E = (1/2) Mv^2 + (1/2) kx^2.
Therefore,
     E = (1/2)(0.2)3^2 + (1/2) 60 (0.1)^2 = 0.9 + 0.3 = 1.2 J
     E = (1/2)MV^2 implies 1.2 = 0.1 V^2.
Hence V = \sqrt{12} = 3.46 m/s.
```

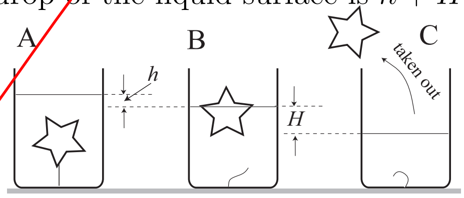
4. A pendulum made of a small mass and a string of length 1 m is gently released from its initial position with a displacement angle of $5°$. After how many seconds does the speed of the mass reach its maximum for the first time? [5]

```
This Pendulum starts from the maximum displacement state.
The state with the max speed is realized when the mass is at the
lowest point.  It takes one quarter of period to go from the
`leftmost' position to the bottom.
     One period = T = 2\pi \sqrt{L/g}, so
     T/4 = (\pi/2)\sqrt{1/g} = 0.5017 s.
```

Immediately, you must guess that this is smaller than \rho.

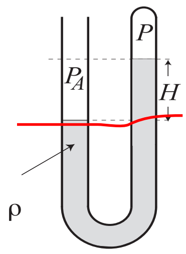
Name: ____________ **Section:** ________ **Score:** ________/20

1. Totally immersed in a liquid of density ρ in a cylindrical container is a uniform object tethered to the bottom of the container as illustrated in A. When the tether is cut, the object bobs up to the liquid surface and stays there as shown in B. The liquid surface goes down by a distance h. When the object is taken out of the container as in C the liquid surface goes down further by a distance H. What is the density of the object in terms of ρ, h and H? (Be sure that the total drop of the liquid surface is $h + H$ in C from A.) [5]




```
To obtain the density of a uniform object, we need its mass M and
volume V.
We can find V from C and A: V = (h + H)A.
Comparing B and C, we know the volume below the surface in Fig. B: HA,
which is the volume of the displaced liquid by the object.

Now, Mr Archimedes tells us that the buoyancy force acting on the object
is \rho times the volume of the displaced liquid times g = \rho HAg.  This
supports the mass of the object M, so \rho HA = M.
Therefore, the density = \rho HA/(h + H)A = \rho H/(h+H).
```

2. One end of a U-tube is closed, and a liquid of density ρ partially fills it as shown in the figure. The closed end has a vapor pressure P and the other end is open to the air (the atmospheric pressure is P_A). The liquid surface in the closed branch is higher by H than that in the open branch. Find $P_A - P$ in terms of ρ, H and g, the acceleration due to gravity. [5]

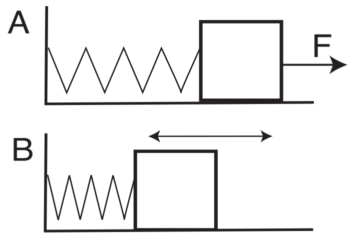


Read the explanation for A1

```
Compare the pressure at the red line:
     Left: Pressure = P_A,
     Right: Pressure = P + H\rho g.
Therefore,
     P_A - P = H\rho g.
```

(**3** and **4** on the next page)

3. A block of mass M is attached to a wall by a spring with spring constant k. Initially, a force of magnitude F pulls the mass to the right as in A. The block is at rest when the force is turned off. What is the maximum magnitude of the acceleration a_M of the block when it oscillates as in B? Assume that the floor is horizontal and frictionless. [5]




```
When F is turned off, the displacement decreases, so the starting
position gives the amplitude.  There, the acceleration is max.
At this moment, the force the spring exerts on the block is F.
Therefore,
     a_M = F/M.
```

4. A pendulum is made from a string of length 1 m and a small weight of mass m attached to its end. If we wish to have the same period of the pendulum on Mars as on Earth, what is the required length of the string on Mars? The acceleration due to gravity on Mars is 38% of that on Earth. [5]

```
The period is
     T = 2\pi \sqrt{L/g}.
Therefore, to keep T the period L/g must be the same:
     L'/g' = 1/g,
so
     L' = g'/g = 0.38 m.
```