

HW 1 due at 9 am on Feb 8, 2012.

1. The fundamental thermodynamic equation of state for a substance is a relation between entropy S and the thermodynamic coordinates, E (internal energy) and work coordinates such as volume V and magnetization M , and number of particles N . Find at least three relations from the five candidates below that violate some of the fundamental principles of thermodynamics and state why they are not realizable as the equations of state for stable substances. All the Greek letters denote positive constants.

- (a) $S = \alpha(NVE^2)^{1/4}$,
- (b) $S = \alpha(EV)^{1/2}e^{-\gamma V/N}$,
- (c) $S = \alpha V^2/E$,
- (d) $S = \alpha E \log(EV/N^2)$,
- (e) $S = \alpha V \log(EV/N)$.

2. [Fun problem]

(1) [Monty Hall Problem] There are three boxes, A, B, C, of which one contains a prize. The player is asked to choose one box. Then, Monty opens one of the empty boxes and asks the player whether she would switch the choice. Should the player switch the box? (This is an extremely famous problem, so you can find the answer somewhere (say Wikipedia). You may copy it, if you understand it, but respect the copyright.)¹

(2) There are two kittens. At least one of them is a male. What is the probability that the other kitten is a male? What is the probability that the other kitten is a female? (Assume that the sex ratio of kittens is 1 to 1.)

3. There is a d -dimensional (classical) ideal gas consisting of N particles whose (one-particle) energy-momentum relation reads $\epsilon = c|\mathbf{p}|^\alpha$, where α and c are positive constants. Find the equation of state (i.e, PVE relation), C_V (the constant volume heat capacity) and C_P (the constant pressure heat capacity). Use dimensional analysis and avoid any integration. [$d = 3$, $\alpha = 2$ is the most popular case as you know.]

4. Demonstrate that Maxwell's relation can always be written in the following form:²

$$\frac{\partial(X, x)}{\partial(y, Y)} = 1.$$

5. What is the sign of

$$\left. \frac{\partial S}{\partial F} \right|_L$$

for rubber? You may assume $(\partial S/\partial L)_T < 0$. First, guess your answer with an intuitive (microscopic) explanation.

¹If you feel the answer not very intuitively acceptable, think of the 10 thousand box case. After you choose one box, Monty opens all but one remaining boxes. What will you do sensibly?

²One way is to check all the possible cases. Is there a better way? [This is not required.]