Decoherence

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Basics

18 Kochen and Specker [38] demonstrated that any hidden-variables theory must necessarily be contextual: The particular value of a physical quantity ascribed to a quantum system (by means of the hidden variables) will in general be dependent on the measurement context, i.e., in which particular manner this value is eventually measured. For example, depending on which other observables are co-measured on the system, the system would possess different values of a particular observable. Contextuality therefore forces us to relinquish the key idea motivating the hidden-variables program in the first place: That the physical world is independent of any measurements performed on it, and that a measurement simply reveals the preexisting value (determined by the values of the hidden variables) of a physical quantity.

19 Thus the widely accepted Born-Pauli interpretation of the wave function is neither purely ontological (in the sense of viewing quantum states as directly representing physical reality) nor simply epistemic (i.e., as representing but the lack of our subjective knowledge).

21 Superposition: ensemble interpretation leads to the problem of postselection.
Experimental verification of superposition:
25 Direct measurement: this is to carry out projective measurement. Stern-Gerlach
Interference experiments: double slit, spatial (the usual one) or temporal (Ramsey inter-
27 ferometry, cf neutrino oscillation). The Copenhagen interpretation is untenable, because it
places the Heisenberg cut: the gap between macro and micro.

28 Entanglement of two systems. If the state of the composite system cannot be written as a direct product of the states of its subsystem, the state is called entangled. Example:

$$\frac{1}{\sqrt{2}}(|0A\rangle|0B\rangle \pm |1A\rangle|1B\rangle). \tag{0.0.1}$$

The term was introduced by Schrödinger in 1935.

31 No-signaling theorem: generalization of EPR.

41 Degree of mixedness One is $Tr\rho^2 \leq 1$ The other is von Neumann entropy $S = -Tr\rho\log\rho$. Note that ρ cannot generally reconstruct the classical probability distribution of pure states; it depends on the basis since

$$\rho = \frac{1}{2}(|0x\rangle\langle 0x| + |1x\rangle\langle 1x|) = \frac{1}{2}(|0z\rangle\langle 0z| + |1z\rangle\langle 1z|). \tag{0.0.2}$$

43 Physical vs statistical ensemble: Physical ensemble is a direct product, so it is quite different from a single system mixed density matrix; they agree only in the sense of ensemble average.

50 The measurement problem (and the problem of the quantum-to-classical transition) is composed of three parts:
1. The problem of the preferred basis. What singles out the preferred physical quantities in nature, e.g., why are physical systems usually observed to be in definite positions rather than in superpositions of positions?
2. The problem of the nonobservability of interference. Why is it so difficult to observe

quantum interference effects, especially on macroscopic scales?
3. The problem of outcomes. Why do measurements have outcomes at all, and what selects a particular outcome among the different possibilities described by the quantum probability distribution?

It is fair to conclude that decoherence has essentially resolved the first two problems. The role played by decoherence in addressing these two issues is rather undisputed.

von Neumann scheme for ideal quantum measurement: System S and measuring apparatus A

$$|Si\rangle|Ar\rangle \to |Si\rangle|Ai\rangle \tag{0.0.3}$$

where Ar is the 'ready state,' and Ai is the pointer state i. Thus, the ideal quantum measurement reads, when $|\psi\rangle = \sum c_i|Si\rangle$

$$|\psi\rangle|Ar\rangle \to \sum_i c_i|Si\rangle|Ai\rangle. \tag{0.0.4}$$

Entanglement is created dynamically.

53 What determines $\{|Ai\rangle\}$? This is the preferred basis problem. There are two constraints:
(1) These state must be orthogonal to be classically distinguished.
(2) The corresponding $|S_i\rangle$ must also be orthonormal (because they are eigenstates of observables).

54 Notice that this decomposition is not unique as illustrated by

$$|\Psi\rangle = \frac{1}{2}(|S0x\rangle|A0x\rangle + |S1x\rangle|A1x\rangle) = \frac{1}{2}(|S0z\rangle|A0z\rangle + |A1z\rangle|A1z\rangle). \tag{0.0.5}$$

Notice that the above two situations requires a physical rotation of the magnetic field and would thus correspond to a physically different setup. For each situation there is a preferred observable.

The existence of such a "preferred observable" (or of a "preferred basis") is thus not explained by the final system-apparatus state arrived at through a von Neumann measurement. This problem of the preferred basis was first cleanly separated out from the problem of wave-function collapse and the intimately related problem of outcomes (see Sect. 2.5-4 below) by Zurek [8].

56 Traditionally, nonobservability of interference was ascribed to the insufficiency of resolution (due to smallness of de Broglie wavelength for macroscopic systems). it is possible to observe spatial interference patterns for mesoscopic molecules in experimental setups that are similar in spirit to the double-slit experiment but circumvent the obstacle of having to manufacture microscopic slits . Yet, when certain experimental parameters unrelated to the diffraction process are changed (for instance, the density of air surrounding the diffracted molecules), the interference pattern is observed to decay. There are other factors that prevent us from observing the interference pattern.

A fortiori, from the final state (0.0.4) of the von Neumann measurement scheme it follows that superpositions involving macroscopic measurement devices should be ubiquitous in nature. Why, then, do we not seem to observe interferences between different pointer positions of the apparatus in the everyday world around us? This is the problem of non-observability of interference.

The problem of outcomes is despite the superposition in (0.0.4) consists of two parts:

(i) Why do we have a definite outcome?
59 (ii) How is this out come selected? The problem of outcomes is rooted in the question of what actualizes a particular result in a probabilistic theory.

62 Double slit Bohr-Einstein debate; The measurement of the momentum of the slit washes way the interference pattern. A more careful analysis by Wooters and Zurek:
Let $|\psi_1\rangle$ is the state going through the slit 1, and $|\psi_2\rangle$ the slit 1. The slit detector state is $|r\rangle$ (ready state) and $|1\rangle$ or $|2\rangle$. Then,

$$\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)|r\rangle \to \frac{1}{\sqrt{2}}(|\psi_1\rangle|1\rangle + |\psi_2\rangle|2\rangle) \tag{0.0.6}$$

The particle density matrix is (Tr_s is the partial trace wrt the slit detector)

$$\begin{aligned}
\rho_p &= \frac{1}{2}Tr_s(|\psi_1\rangle|1\rangle + |\psi_2\rangle|2\rangle)(\langle\psi_1|\langle 1| + \langle\psi_2|\langle 2|) && (0.0.7)\\
&= \frac{1}{2}Tr_s(|\psi_1\rangle|1\rangle\langle\psi_1|\langle 1| + |\psi_2\rangle|2\rangle\langle\psi_1|\langle 1| + |\psi_2\rangle|2\rangle\langle\psi_2|\langle 2| + |\psi_1\rangle|1\rangle\langle\psi_2|\langle 2|) && (0.0.8)\\
&= \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_1|\langle 1|2\rangle + |\psi_2\rangle\langle\psi_2| + |\psi_1\rangle\langle\psi_2|\langle 2|1\rangle) && (0.0.9)\\
&= \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|) + Re(|\psi_2\rangle\langle\psi_1|\langle 1|2\rangle) && (0.0.10)
\end{aligned}$$

If 1 and 2 are orthogonal, we have a perfect info about the slit, and the last term disappears. If 1 and 2 are identical, no slit information is obtained, and we recover full interference pattern. Simultaneously partial information may be obtained.

66 Let us assume that a macroscopic object has two states $\psi_{1,2}$. Many fields and molecules interact with it and 'carry away' information about the states. Depending of the states, the post-scattering state of the environment are almost distinct (orthogonal)[1] and $|E1\rangle$ and $|E2\rangle$. Quite analogously, the macrostate density matrix reads

$$\rho_M = \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|) + Re(|\psi_2\rangle\langle\psi_1|\langle E1|E2\rangle) \tag{0.0.11}$$

68 The last term is almost zero. Because of this very large number of environmental degrees of freedom interacting with the system and our inability to directly manipulate them, the creation of system-environment entanglement is virtually impossible to undo in practice. Thus the environment-induced loss of local phase coherence, i.e., of the well-defined phase
69 relations between the components in the superposition necessary for the observation of interference effects, is usually irreversible for all practical purposes.

This practically irreversible delocalization of phase relations into the composite system-environment state induced by inevitable and ubiquitous environmental monitoring constitutes precisely the process of decoherence. It leads to effectively nonunitary dynamics for the local system that may manifest themselves (for example) in the decay of interference patterns. Environmental monitoring and the resulting decoherence processes therefore provide a solution to the problem of the nonobservability of interference. All physical systems encountered in nature are open quantum systems that interact strongly with their surroundings. These surroundings continuously acquire information about the system, leading to a

[1] We may assume $|E1\rangle = \prod |ei1\rangle$ (numerous subsystems), so even if $|\langle e1i|e2i\rangle| \sim 1$, $\langle E1|E2\rangle \simeq 0$.

constant "leakage" of coherence from the system into the environment.
Warning Do not interpret (0.0.11) as the system state is either 1 or 2. It is fully superposed still.

70 A large part of decoherence study is realistic models of the decoherence process itself: how $\langle E1|E2\rangle$ decays to zero.
73 If a state is least entangled with the environment, it is most immune to decoherence, and the state is easy to measure. The sate selected by the stability criterion in this sense is called the pointer state.

76 To understand decoherence dynamically, we need a system-environment interactions Hamiltonian. In the quantum-measurement limit the total Hamiltonian is dominated by the interaction Hamiltonian. To prevent decoherence, entanglement should not occur, so we demand

$$e^{-iH_{int}t}|Si\rangle|E\rangle = |Si\rangle|E(t)\rangle. \tag{0.0.12}$$

77 The pointer state Si must be an eigenstate of (the system subspace part of) H_{int}; the pointer state must be stationary under interaction. The observable that give pointer states as its eigenkets are called pointer observable. Such an observable must be commutative with H_{int}.
78 In many cases $H_{int} = \sum_\alpha \hat{S\alpha} \otimes \hat{E\alpha}$, where $\hat{S}$ is the observable, and $\hat{E}$ is the environmental H. This describes simultaneous monitoring of $S\alpha$ by the environment. In this case, if $|Si\rangle$ is
79 a simultaneous eigenstate of all $\hat{S}\alpha$, obviously

$$e^{-iH_{int}t}|Si\rangle|E\rangle = |Si\rangle|E(t)\rangle \tag{0.0.13}$$

holds. Notice that in this case there is no piece acting jointly on the system and environment.

81 If the environment is slow compared with the system (quantum limit of decoherence), the system can monitor only slow variables or only the constants of motion of the system. That is, only the system energy. Energy levels become dominant observable, not because the system is isolated, but because energy is continuously monitored by the environment.
82 General theoretical strategy to select preferred sate by the interaction with the environment is to monitor von Neumann entropy and choose the least entropy increasing as preferred states.

83 System-environment interaction Hamiltonians frequently describe a scattering process of surrounding particles (photons, air molecules, etc.) interacting with the system under study. Since the force laws describing such processes typically depend on some power of distance (such as r^{-2} in Newton's or Coulomb's force law), the interaction Hamiltonian will usually commute with the position operator. According to the commutativity requirement, the pointer states will therefore be approximate eigenstates of position. The fact that position is typically the determinate property of our experience can thus be explained by referring to the dependence of most interactions on distance. This origin of the special role of position in the quantum-to-classical transition was clearly pointed out and analyzed for the first time by Zurek [8, 9, 13]. Subsequently, the scattering model of Joos and Zeh [7] showed directly how surrounding photons and air molecules continuously measure the spatial structure of small objects such as dust particles, leading to rapid decoherence into an (improper) mixture of narrow position-space wave packets (see Chap. 3).

Chiral molecules such as sugar are always observed to be in chirality eigenstates (left-handed and right-handed). This can be explained by the fact that the distinct spatial structure of these molecules is continuously monitored by the environment through scattering processes. This environmental monitoring gives rise to a much stronger coupling to the "outside world" than could typically be achieved by a measuring device that was intended to measure, say, parity or energy. Furthermore, any attempt to prepare such molecules in energy eigenstates will lead to immediate decoherence into the environmentally stable chirality eigenstates.

84 The quantum limit of decoherence [103] results in the environment-induced selection of energy eigenstates with interference between different energy eigenstates being continuously suppressed due to the environmental monitoring of the energy of the system.

[**C**] Superposition of constants of motion does not occur generally.

Superselection of charge is due to the interaction of the charge with its own Coulomb (far) field.[2] This field plays the role of the environment, leading to immediate decoherence of charge superpositions into (improper) mixtures of charge eigenstates.

To summarize, we have distinguished three different cases for the type of preferred pointer states emerging from interactions with the environment:

1. The quantum-measurement limit. When the evolution of the system is dominated by H_{int}.
2. The quantum limit of decoherence. When the environment is slow and the self-Hamiltonian dominates the evolution of the system.
3. The intermediate regime: this is compromise of 1 and 2.

85 A characteristic feature of classical physics is the fact that the state of a system can be found out and agreed upon by many independent observers. In this sense, classical states
86 preexist objectively. By contrast, measurements on a closed quantum system will in general alter the state of the system. It is therefore impossible to regard quantum states of a closed system as existing in the way that classical states do.

Since information about the system is encoded in the environment, we can acquire this information without having to directly interact (and thereby disturb) the system itself.
How, and which kind of, information is both redundantly and robustly stored in a large number of distinct fragments of the environment in such a way that multiple observers can retrieve this information without disturbing the state of the system, thereby achieving effective classicality of the state? The research of the Los Alamos group into answering this question is carried out under the headings of the "environment as a witness" program (the recognition of the role of the environment as a communication channel) and quantum Darwinism (the study of what information about the system can be stably stored and proliferated by the environment).

88 Departing the closed-system view and of describing observations as the interception of information stored in the environment represents a very promising candidate for a purely QM account of classicality.

89 Simple model of decoherence

Interacting spin system interpreted the 'central spin' to be the system and the rest, the environment.

$$H = H_{int} = \sigma_z \otimes \sum_i g_i \sigma_{iz} \equiv \sigma_x \otimes \hat{E}. \tag{0.0.14}$$

The eigenstates of this Hamiltonian has the form $|u\rangle|n\rangle$ or $|d\rangle|n\rangle$, where $|u\rangle$ and $|d\rangle$ are the central spin up or down, and $|n\rangle$ is the environment n spins up state. Any pure state of this system can be written as a linear combination of these basis vectors

$$|\Psi\rangle = \sum_n (c_n|u\rangle + d_n|d\rangle)|n\rangle. \tag{0.0.15}$$

[2] Giulini, Kiefer, and Zeh [85] (see also [83])

The initial state is

$$|0\rangle = (a|u\rangle + b|d\rangle)\sum_n c_n|n\rangle. \tag{0.0.16}$$

and

$$|t\rangle = e^{-iHt}|0\rangle = a|u\rangle|E_u(t)\rangle + b|d\rangle|E_d(t)\rangle. \tag{0.0.17}$$

91 where

$$|E_u(t)\rangle = |E_d(-t)\rangle = \sum_n c_n e^{-ite_nt}|n\rangle \tag{0.0.18}$$

with $e_n = \sum_i (-1)^{s_i} g_i$ (s_i is the z-component of the i-th spin). The decoherence factor $r(t)$ is

$$r(t) = \langle E_u(t)|E_d(t)\rangle = \sum_n |c_n|^2 e^{-2ie_nt}, \tag{0.0.19}$$

since $r(t) \to 0$ implies total decoherence. If there are N environmental spins, there are 2^N
92 terms. This is almost a RW on $\boldsymbol{C}$, so $r^2(t)$ scales as 2^{-N}. Also $r(t) \sim e^{-\Gamma t^2}$ can be shown; Γ depends on g_i and the intial condition of the environment.[3] The Poincare time is extremely long.

93 If dissipation is absent or negligible, we must not conclude that there is no (or only negligible) interaction between the system and its environment. Even if a particular type of interaction does not lead to dissipation, it will in general still result in decoherence, which is a pure quantum effect. That is, the environment may in general obtain which-path (or, more generally, which-state) information without absorbing any energy from the system.

If both exists, the time scales are vastly different.

$$\tau_r/\tau_d \sim (\Delta x/\lambda_{dB})^2 \gg 1 \tag{0.0.21}$$

usually, where the macrosuperposition of objects is described by two different positions a distance Δx apart.

95 Do not confuse the noise effect and decoherence. Consider superposition of pure states with a random phase

$$|\psi_i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi_i}|1\rangle \tag{0.0.22}$$

and make the ensemble density operator

$$\rho = \frac{1}{N}\sum_N |\psi_i\rangle\langle psi_i| \tag{0.0.23}$$

97 The cross term is with a random sum $(1/2N)\sum e^{-i\phi_i t}$, which is very small for a large N. This is not due to decoherence., but from the density-matrix point of view not distinguishable (fake decoherence). Noise—the addition of random fluctuations to the Hamiltonian of the system—does not create any system-environment entanglement and can be completely
98 undone (at least in principle) by local operations. Quantum decoherence is generally nonlo-

[3]If all $g_i = g$ and the intial condition is one of the eigenstates,

$$r(t) = \cos^N(gt) \tag{0.0.20}$$

cal and its time scale is very short.

101 Decoherence derives from the presupposition of the existence and the possibility of a division of the world into "the system" and "the environment." "Our experience of the classical reality does not apply to the universe as a whole," (Zurek).

102 By definition, the universe as a whole is a closed system, and therefore there are no "unobserved degrees of freedom" of an external environment. There exists no general criterion for how the total Hilbert space is to be divided into subsystems: "What are the systems?"

Technial notes:

104 (1) Schmidt base: If a composite system consists of system A and B. Then its direct product Hilbert space can have a 'diagonal basis' such that $\{|ai\rangle\}$ and $\{|bi\rangle\}$ are bases of A and B, respectively, and $\{|ai\rangle|bi\rangle\}$ is a basis of AB. A given pure state may always be written as

$$|\Psi\rangle = \sum \sqrt{p_i}|ai\rangle|bi\rangle \tag{0.0.24}$$

by twiddling the bases, where $\sum_i p_i = 1$. For this state reduced density matrices read

$$\rho_A = \sum_i p_i|ai\rangle\langle ai|, \quad \rho_B = \sum_i p_i|bi\rangle\langle bi|. \tag{0.0.25}$$

107 (2) Wigner representation: it could be negative. Despite this, W-distribution can give very suggestive pictures as in the case of the double slit experiment.

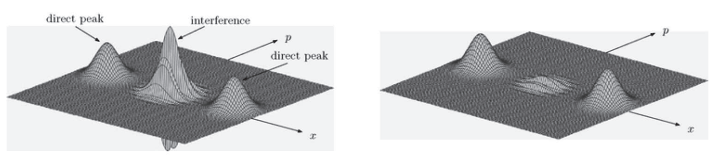

Figure 0.0.1: With the interaction with the environment the central interference peak dissipates. [Fig. 2.8, 2.9]

109 (3) Purification theorem: Any mixed state density matrix may be understood as a reduced pure density matrix of a larger state space. This means that at $t = 0$ we may always regard the environment state pure.

110 (4) Kraus operator formalism:

Decoherence Detailed Models

115 Together with the emission of thermal radiation (see Sect. 6.2.5), environmental scattering is the dominant and ubiquitous process for decoherence in the macroscopic domain. Air molecules, light (optical photons), background radioactivity, cosmic muons, solar neutrinos, and even the 3 K cosmic background radiation present everywhere in the universe continuously monitor the position of the quantum system of interest.

118 Over the past few years a rapid progress in experimental techniques has enabled researchers to make very precise measurements of decoherence time scales even for mesoscopic and macroscopic objects.

135 Decoherence time scale (in s) due to thermal photons or air molecules

Environment	dust grain (10 μm)	large molecule (10 nm)
3 K background	1	10^{24}
room temp photon	10^{-18}	10^{6}
best lab vacuum	10^{-14}	10^{-2}
air at normal pressure	10^{-31}	10^{-19}

However, the calculation uses the assumption that there is no recoil, so for large molecules some caution may be required.

136 The wave function of an electron with an initial spatial width of 1 Å would very quickly become completely delocalized under purely unitary time evolution, reaching a width of about $\Delta x = 10^8$ cm within a second.

Our numerical estimates demonstrate that localization of such a spread-out wave packet would occur within less than a second even if only the environment of solar neutrinos was taken into account. Thermal photons, on the other hand, would lead to an extremely short decoherence time of about 10^{-16} s for the electron described by this delocalized wave packet.

6. Observing decoherence in action

265 Decoherence due to thermal radiation: heated buckyball decohere more quickly.

272 Squid: 10^9 Cooper pairs. a few tens of ns may be the decoherence time scale. To make
281 a quantum computer 3 orders longer time scale is at least needed. Intrinsic defects in the Josephson junction is the main source of decoherence.

283 Atomic gas BEC: the main source of decoherence is the noncondensates