The Monty Hall Problem

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1. Ancestral Monty

5 Probablity is hard[1]

(i) Birthday problem

7 (ii) False positive in medical testing: disease 1/1,000 are afflicted. There is a test for the disease which is 95% accurate. It never gives a false negative. You have tested positive. What is the probability that you actually have the disease? Out of 1,000, about 50 people get false positive, and only 1 true positive on the average, so the probability is about 2%. That is, 0.1% has been 'improved' to be 2%.

10 Russel expressed the paradox at eh heart of probability by asking rhetorically, "How dare we speak of the laws of chance? Is not chance the antithesis of all law?"[2]

13 Bayes: how to update a prior probability assessment in the face of the new evidence.

14 The Bertrand box paradox: three boxes: the first containing 2 gold coins, the second 2 silver coins, and the third both. Bertrand asks: remove one coin from a box (without checking it); what is the probability of the remaining coin to be gold? 1/2? What if we know the removed one was gold? (2/3 for gold in this case). Similarity to this to the Monty problem is clear.

16 Three prisoners. Choose 2 out of three elements A, B, and C. If A is not chosen, A is told that B or C is chosen with equal probability. If A is chosen, the other choice is told. A was told that B would be chosen. What is the probability for A to be chosen? ($1/3\times 1/2+2/3\times 1/2 = 2/3$, no change) How about the probability for C to be chosen? ($2/3 \times 1/2 = 1/3$).

20 Monty Hall Problem: A, B, C three boxes of which one contains a prize. The player is asked to choose one box. Then, Monty opens one of the empty boxes. Should the player switch the box?

[C] Note that Monty's choice is not a random choice, so the outcome is not due to neutral operation to the probability of the event.

23 Marylin vos Savant's explanation: suppose there are 10,000 boxes and the Monty empties all the boxes not chosen but one. You should choose that box! Notice that the probability for the remaining box to contain the prize + the first box to contain the prize = 1. The overall success rate with witching is $-1/3 \times 1/2 + 2/3 = 1/2$.

26 There are two puppies; at least one of them is a male. What is the probability

[1] J. Roesnthal, *Struck by Lightening:the curious world of probabilities* is recommended.

[2] For the history of probability see I. Hacking, *The emergence of probability*; F F David, *Games, Gods, and Gambling.*

that the other one is also a male? 1/3 (because mm, fm, mf).