Philosophical Introduction to Probability

CSLI Lecture Notes number 167
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CSLI Publication (2005)

2. The laws of probability

53 every interpretation embodies a particular definition of probability, together with a method of fixing priors.
According to all interpretations probability has to be a real number in the interval 0-1, and to obey the rules of addition and multiplication.

54 Decisive merit of Kolmogorov's axiomatization is precisely that of tracing a clearcut boundary betwee the mathematical properties of probability and its interpretations.
cf Roeper and Leblanc, *Probability Theory and Probability Logic* (U Toronto, 1999).

66 Objection to Laplace: It is impossible to know all the cases, and the set of 'equally likely cases.'

67 Bertrand's paradox: if there are infinitely many possibilities, principle of indifference could lead to contradictions.

68 Wesley Salmon[1]
"Suppose we know that a car has taken between 1 and 2 minutes to traverse one mile." What is the probability of the car take less than 1.5 minutes? "Our data, however, can be expressed in another way. wWe know that the average speed for the trip was between 60 and 30 mph." What is the average speed?"
This is a general difficulty when nonlinear relations exist.

4. The frequency interpretaion

75 Venn's radically empirical attitude is summarized by saying that 'our ultimate reference is always to facts. We start from them as our data, and reach them again eventually in our results whenever it is possible. Starting from the experienced, probability leads to infer non experienced facts.

78 While affirming the idea that 'probability is a science of inference about real things', Venn strongly opposes the view that probability is a measure of belief.

81 The most perspicacious version of the frequency theory of probability was provided by von Mises.

83 von Mises says that talking of the probability of single events 'hs no meaning' (= metaphysics).

85 While regarding randomness (of lawlessness) as an essential feature of probabilistic sequences, von Mises reaffirms the theoretical priority of this notion over that of probability.

87 Von Mises' definition of randomness rigorously restates the long-standing idea that events are random when they are unpredictable and cannot be accounted for in causal terms. It should not pass unnoticed that while defining randomness is terms of insensitivity to *all* possible place selections, von Mises embraces an absolute, unrestricted notion of randomness. This choice is philosophically motivated by an urge to secure an objective foundation to probability. However, soon after it was proposed by von Mises, his theory of randomness raised serious objections.... Anticipating what will follow, it can be added that the difficulties

[1] *Foundation of Scientific Inference* (U of Pittsuburgh P, 1966).

affecting von Mises' approach shed doubts on the whole project of defining randomness in absolute terms.

5. The propensity interpretation

105 Peirce: probability doe not simply refer to past occurrences, and cannot therefore be calculated simply by taking the ratio between the number of occurrences of the event and the number of observed cases. Probability is rather 'the ratio that there *would be* in the long run'. $\cdots$ It is because of the stress he puts on the *would-be*, or on other words describing the dispositional character of probability, that Peirce can be seen as anticipating the propensity theory.

108 Popper's falsificationist methodology of 'conjectures and refutations' includes a notion of corroboration, defined in terms of resistance to sever tests.

110 Popper oscillates between two different standpoints, namely a 'long run' and 'single case' propensity interpretation. This was pointed out by Gillies.

Propensities are defined as 'weighted possibilities' and see as measurable expressions of the tendency of a possibility to realize itself upon repetition. $\cdots$ But now the emphasis is on single arrangements, rather than on sequences of generating conditions, for Popper claims that 'every experimental arrangements (and therefore *every state of a system*) generates propensities'. In some cases, namely when they are referred to mass phenomena of to repeated experiments, propensities can be measured by means of frequencies.

132 Is chance objective? (Should be conclude that chance is a product of human ignorance, as Laplace put it?)

133 Poincaré's thoughts about chance impressed von Mises, who explicitly opened the door to indeterminism, and therefore to the existence of chance phenomena in nature. However, von Mises' indeterminism does not seem to imply any overarching metaphysical hypothesis about the world; it looks rather like an epistemological attitude, meant to accommodate within a unified framework all scientific knowledge.

6. The logical interpretation

135 The logical interpretation of probability can be seen as a natural development of the idea that probability is an epistemic notion, pertaining to our knowledge of facts, rather than to facts themselves. "As our knowledge or our hypothesis changes, our conclusions have new probabilities, not in themselves, but relatively to these new premisses."[2]

146 The theory of probability is conceived by Keynes as a branch of logic, more precisely as that part of logic which deals with arguments that are not conclusive. but can be said to have a greater or less degree of inconclusiveness.

147 Keyens wants to develop a theory of the reasonableness of degrees of belief on logical grounds. According to him, the theory of probability as a logical relation: 'is concerned with the degree of belief which it is *rational* to entertain in given conditions, and not merely with the actual beliefs of particular individuals, which may or may not be rational."

148 The measurement of probability rests on the principle of insufficient reason: 'In order that

[2]R B Braithwaite, 'Keynes as a philosopher' in M Keynes ed: *Essays on John Maynard Keynes* (Cambridge UP 1975) p7

numerical measurement may be possible, we must be given a number of *equally* probable alternatives.'

152 Ramsey's criticism: 'there really do not seem to be any such things as the probability relations he describes.'

Keynes wrote an obituary, containing an explicit concession to Ramsey's criticism. There he says: "a priori probability is part of our human outfit, perhaps given us merely by natural selection, analogous to our perception and our memories rather than to formal logic. So far I yield to Ramsey—I think he is right."[3]

165 Carnap was inspired by an unwavering faith in the powers of formal logic on the one side, and of experience on the other, in compliance with the logical empiricist creed.

His *Logical Foundation of Probability* contains: "(4) the so-called frequency concept of probability, as used in statistical investigations, is an important scientific concept in its own right, but it is not suitable as the basic concept of inductive logic."

166 Carnap admits two notions of probability, one logical and one statistical.

167 Carnap points out that both concepts of probability has an objective import. As a matter of fact, in the Forties Carnap seems unable to even conceive of a subjective notion of probability.

169 Probability$_1$ 'has its place in inductive logic and hence in the methodology of science', probability$_2$ 'in mathematical statistics and its applications'.

171 In order to measure degree of confirmation, one starts by assigning a wright to state descriptions.

177 To the question what reasons can be given for accepting the axioms of inductive logic, Carnap answers that 'the reasons are based upon our intuitive judgments concerning inductive validity, i.e., concerning inductive rationality of practical decisions(e.g., about bets)'

178 Jeffreys, after studying under Carnap in Chicago and later publishing his last manuscripts, came to the conviction that Carnap's conception of logic of confirmation as a blend of a purely logical component and a purely empirical element, should be superseded by a more eclectic approach, close to Bruno de Finetti's subjective Bayesianism.

179 Jeffreys is an out and out Bayesian, who used to say that Bayes' theorem 'is to the theory of probability what Pythagoras' theorem is to geometry.'

183 While embracing epistemic notion of probability, Jeffreys moves a severe criticism to the frequency theory. His criticism is directed to infinite ensemble sets, and 'the Venn limit' as useless.

184 Jeffreys claims to be 'inclined to think that there may be such a thing as intrinsic probability.'

Jeffreys' epistemology is rooted in a phenomenalisitc view of knowledge, that he claims to derive from Mach and Pearson.

The author's attitude can be described as constructivist, in the sense that for him such notions as 'objectivity' and 'reality' are established by inference from experience. This requires the adoption of statistical methodology, which is the core of scientific method.

185 The deterministic version of the principle of causality is thereby discarded, for 'exact causality in this sense remains a hypothesis; the claim that it is a result of experience is simply false. [...] It expresses a wish for exactness, which is always frustrated, and nothing more.'[4]

[3] Keyens, *The collected writings of John Maynard Keynes* (MacMillan 1972) (original in 1930) p339.

[4] H Jeffreys, Scientific method, causality , and reality' Proceedings of the Aristotelian Society, New Series XXXVII (1937) p63-4.

Jeffreys' position regarding scientific laws, reality and causality seems to be inspired by the same kind of pragmatism underpinning Ramsey's views on general propositions and causality, the main difference being that Ramsey's approach is more strictly analytic, whereas Jeffreys grounds his arguments on probabilistic inference statistical methodology alone. Jeffreys' perspective is close to subjectivism also in other respects: its constructivism, the conviction that science is fallible and that there is continuity between science and everyday life, and last but not least, the admission that empirical information can be 'vague and half-forgotten', a fact that 'has possibly led to more trouble that has received explicit mention.[5] as a matter of fact, the pragmatic attitude take by Jeffreys towards epistemology is somehow at odds with his definition of probability as degree of rational belief uniquely determined by experience, and with the idea that the evaluation of probability is an objective procedure, whose application to experimental evidence obeys rules having the status of logical principles.

7. The subjective interpretation

191 Borel's main discontent with Keyens' work lies with the conviction that it overlooks the application of probability to science, to focus only the probability of judgments.

194 de Finetti praises Borel for pleading the idea that probability must be referred to the simple case, and for holding that this kind of probability is always measurable sufficiently well by means of the betting method. At the same time de Finetti expresses strong disagreement with the eclectic attitude take by Borel, more particularly with his admission of an objective meaning of probability in addition to the subjective–a position that, as we shall see, de Finetti always rejected.

195 Ramsey's work prepares the ground for modern subjectivism, sometimes also called 'personalism.' See *Foundations of Mathematics and Other Logical Essays* (ed B Braithwaite).

196 For Ramsey, probability is a degree of belief, and probability theory is a logic of partial belief. The notion of 'degree of belief is taken as a primitive notion, which admittedly 'has no precise meaning unless we specify more exactly how it is to be measured.'
A first option to achieve this purpose is the method of bets. However, this is not accurate, if we consider, eg the problem of diminishing marginal utility of moeny. etc.

In order to avoid such difficulties, Ramsey adopts a different method based on the notion of 'preference.' This is basically optimization (so measurable). Ramsey's characterization of degree of belief is just identical to probability (i.e., 'linear relation' between the belief and the action is assumed.)

199 The crucial link between probability and degree of belief provided by consistency (or coherence—to use more widespread term) is the cornerstone of subjective probability.

An important consequence of the adoption of a notion of probability in terms of coherent degree of belief is that Ramsey does not need to ground his own theory on the principle of indifference. ... This is a decisive step in the moulding of the modern subjectivism.

201 Ramsey's way of lookiing at the relationship between logic and probability is utterly different from that of Keynes. He distinguishes between 'lesser logic' (= Wittgensteinian tautological formal logic) and 'larger logic' (= the logic of discovery, or inductive logic). The latter is based on psychology not on logic.
Tractasus 6.363, 6.3631: the process of induction ' has no logical foundation but only a psychological one.' Ramsey praises this.

202 As to the relation between frequentism and subjectivism: partial belief involves reference to

[5] H Jeffreys, *Scientific Inference* (Cambridge UP, 1931;modified edition 1973).

a hypothesis of ideal frequency belief of degree m/n....

203 Ramsey's pragmatical tendency to refer belief to action, and to justify inductive behavior with reference to successful conduct.

To sum up, the relationship between degree of belief and frequency is an open problem within Ramsey's perspective.

204 Ramsey was not a dualist: in the last years of his lie, Ramsey was developing a view of 'chance' and 'probability in physics' fully compatible with his subjective interpretation of probability as degree of belief.

208 Working in the same yeas as Ramsey, but independently, de Finetti forged a similar view of probability as degree of belief, obeying the sole requirement of coherence. To such a definition of probability he added the notion of exchangeability which combined with Bayes' rule , gives rise to the inferential methodology, which is at the root of the so-called neo-Bayesianism.

210 de Finetti put forward an original philosophy of probability , which can be described as a blend of pragmatism, operationalism and what we would today call 'anti-realism.'

de Finetti's philosophical position—labelled by Richard Jeffereys 'radial probabilism'—reaffirms a conception of scientific knowledge as a product of human activity, ruled by (subjective) probability, rather than truth or objectivity.

212 The second step of de Finetti's programme for establishing the subjective interpretation of probability consists in the reduction of objective to subjective probability. This is done by means of what is known as the 'representation theorem. The pivotal notion in this context is that of 'exchangeability,' which corresponds to Carnap's notion of 'symmetry' and johnson's 'permutation postulate.' Summarizing de Finetti, events belonging to a sequence are exchangeable if the probability of h successes in n events is the same, for whatsoever permutation of the n events, and for every n and $h \leq n$.

214 for de Finetti subjective probability, being the expression of the feelings of the subjects evaluating it, is always definite and known.

216 "Objective probability never exists."

217 De Finetti's refusal of objective probability goes hand in hand with his lack of consideration for the notion of 'change' and 'physical probability.'

224 Science is concerned with what 'appears to us.' [6]

235 **Concluding remarks**

while frequentism seems to match the uses of probability in areas like population genetics and statistical mechanics, quantum mechanics clashes with its fundamental assumption, namely the tenet that probability can only refer to population, not to single events. [a stupid comment]

[6] de Finetti,in Scientia LXX pp283-303 (1976).