QUANTUM DARWINISM, CLASSICAL REALITY, and the randomness of quantum jumps

W H Zurek, PT 2014 Oct p44

Interactions with the environment help select preferred states of the system. As it is impossible to follow every variable of the composite system-environment whole, one relies on a reduced density matrix, a statistical description of the system alone. Those preferred states, left untouched by the interaction with the environment, are called pointer. They eventually end up as the eigenstates of the reduced density matrix. states. The environment-driven process that selects pointer states is called decoherence.

Postulate 1 Quantum states correspond to vectors in a Hilbert space.
Postulate 0 [Composition postulate] The state of the composite system is expressed as a vector the direct product space of the component system Hilbert spaces.
Postulate 2 Time evolution is unitary: $|t\rangle = e^{-iHt}|0\rangle$.
Postulate 3 [Repeatability] An immediately repeated measurement yields the same outcome.

Classical repeatability is a given: Measurements reveal classical states, so repeatability follows from their objective existence. Observers cannot reveal an unknown quantum state, but repeatability lets them confirm the presence of known states.

A set of more controversial postulates are:
Postulate 4 [The collapse axiom]
4a: Observables are Hermitian.
4b: The outcome of a measurement must correspond to an eigenstate of the measured Hermitian operator.
Postulate 5 [Born's rule] $p_k = |\langle k|\psi\rangle|^2$. This is the key link between math and phys.

Decoherence leads to environment-induced superselection of preferred states, and so accounts for effectively classical states and the menu of measurement outcomes.

Consider a measurement-like interaction of a system S with a quantum apparatus A. The state of A changes, but to ensure repeatability, the state of S does not:

$$|u\rangle|A_0\rangle \xrightarrow{H_{SA}} |u\rangle|A_u\rangle \tag{0.0.1}$$

Here, the arrows represent the unitary evolution determined by the Hamiltonian H_{SA} describing the system-apparatus interaction. Since this evolution is unitary

$$\langle u|v\rangle\langle A_0|A_0\rangle = \langle u|v\rangle\langle A_u|A_v\rangle \tag{0.0.2}$$

Since the measurement apparatus must be able to distinguish states $|u\rangle$ and $|v\rangle$, $\langle A_u|A_v\rangle \neq 1$. This implies $\langle u|v\rangle = 0$. That is, every different observable result must be orthogonal. Therefore, the observable must be real and normal, or Hermitian. Postulate 4a has been derived. Also the above argument shows that when only a discrete set of states in the Hilbert space is stable, the evolution of the system will look like a jump into one of the states. Although there is no literal collapse of the wavefunction, measurement records will suggest a quantum jump from an initial superposition to one of the stable states or from stable state to stable state.

Orthogonal states that survive multiple confirmations of their identity are selected by their interaction with the apparatus or decohering environment. Their superpositions could persist in isolation but cannot be recorded. Only discrete, stable states can be followed. According to decoherence theory, the ability to withstand scrutiny of the environment defines pointer states.

In microsystems, repeatability is, in fact, rare: Nondemolition measurements are difficult. In the macroworld, however, repeatability is essential for the emergence of objective reality. Macrostates such as records inscribed in an apparatus should persist through many readouts, even as the underlying microstates change. A demonstration such as the one given above shows that stable macrostates must also be orthogonal to accommodate repeatability.[1]Much of axiom 4 follows from the simple and natural core postulates 0-3.
[C] However, here macrostates are not critically considered. It cannot be a single state (even macroscopically) because of the bounded precision of any macroscopic observations. □

Decoherence is the loss of phase coherence between preferred states. It occurs when S starts in a superposition of pointer states, but in the decoherence context, S is "measured" by the environment E:

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|E_0\rangle \xrightarrow{H_{SE}} \alpha|\uparrow\rangle|E_\uparrow\rangle + \beta|\downarrow\rangle|E_\downarrow\rangle = |\psi_{SE}\rangle. \tag{0.0.3}$$

The discussion centered around equation (0.0.2) implies that the unaltered states are orthogonal, $\langle\uparrow|\downarrow\rangle = 0$. Their superposition, upon interacting with the environment, turns into an entangled $|\psi_{SE}\rangle$; neither S nor E retains an individual pure state.

Phases in a superposition matter. In a spin 1/2-like system, $|\leftarrow\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$ is orthogonal to $|\rightarrow\rangle = (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2}$. The phase shift operator $\pi_S(\varphi) = |\uparrow\rangle\langle\uparrow| + e^{i\varphi}|\downarrow\rangle\langle\downarrow|$ leaves $|\uparrow\rangle$ untouched and multiplies $|\downarrow\rangle$ by $e^{i\varphi}$; when $\phi = \pi$, it converts $|\rightarrow\rangle$ to $|\leftarrow\rangle$. In experiments, such phase shifts translate into shifts of interference patterns.

For simplicity, assume perfect decoherence: $\langle E_\uparrow|E_\downarrow\rangle = 0$. In that case, the environment has a perfect record of pointer states. Notice that the phase shift $\pi_S(\varphi)$ acting on an entangled can be undone by $\pi_E(-\varphi) = |E_\uparrow\rangle\langle E_\uparrow| + e^{-i\varphi}|E_\downarrow\rangle\langle E_\downarrow|$ a countershift acting on a distant E decoupled from the system:

$$\pi_E(-\varphi)\pi_S(\varphi)|\psi_{SE}\rangle = \pi_E(-\varphi)(\alpha|\uparrow\rangle|E_\uparrow\rangle + \beta|\downarrow\rangle|E_\downarrow\rangle) = |\psi_{SE}\rangle. \tag{0.0.4}$$

This implies that the phase change imposed by the system can be undone by environment; As phases in $|\psi_{SE}\rangle$ can be changed in a faraway environment decoupled from but entangled with the system, they can no longer influence the state of S; if they could, a measurement of S would reveal that influence and enable superluminal communication. Thus, measurements on the system will not detect a phase shift, as there is no interference pattern to shift. The phase information about S is lost. Information loss was established without reduced density matrices, the usual decoherence tool.[2]

The above view of decoherence appeals to symmetry, an entanglement-assisted invariance, or envariance, of S under phase shifts of pointer-state coefficients. As S entangles

[1]W. H. Zurek, Phys. Rev. A 87, 052111 (2013).
[2]W. H. Zurek, Phys. Rev. 71 71, 052105 (2005); Phys. Rev. Lett. 106, 250402 (2011).

with E, its local state becomes invariant under transformations that affected it before the entanglement.

In laboratory experiments, a system isolated from the environment is first measured by an apparatus A so that system and apparatus entangle. That entangled state $|\psi_{SA}\rangle$ then decoheres as A interacts with E:

$$(\alpha|\uparrow\rangle|A_\uparrow\rangle + \beta|\downarrow\rangle|A_\downarrow\rangle)\,|E_0\rangle \xrightarrow{H_{AE}} \alpha|\uparrow\rangle|A_\uparrow\rangle|E_\uparrow\rangle + \beta|\downarrow\rangle|A_\downarrow\rangle|E_\downarrow\rangle = |\Psi_{SAE}\rangle. \quad (0.0.5)$$

The pointer states $|A_\uparrow\rangle$ and $|A_\downarrow\rangle$ of A, however, are unaffected by the decoherence interaction with E. They retain perfect correlation with S (or an observer, or other systems) in spite of E, regardless of the value of $\langle E_\uparrow|E_\downarrow\rangle$. Stability under decoherence is a prerequisite for effective classicality in our quantum universe: The familiar states of macroscopic objects have to survive monitoring by E and retain correlations.

The decohered SA is described by a reduced density matrix obtained by averaging out the environment. When $\langle E_\uparrow|E_\downarrow\rangle = 0$, the pointer states of A retain their correlations with the measurement outcomes:

$$\rho_{SA} = \alpha^2|\uparrow\rangle\langle\uparrow||A_\uparrow\rangle\langle A_\uparrow| + \beta^2|\downarrow\rangle\langle\downarrow||A_\downarrow\rangle\langle A_\downarrow| \quad (0.0.6)$$

Both $\uparrow$ and $\downarrow$ are present. There is no collapse. The averaging over environmental states is implemented by a mathematical operation called taking a trace—that is, $\rho_{SA} = Tr_E|\psi_{SAE}\rangle\langle\psi_{SAE}|$. However, both the interpretation of ρ_{SA} as a statistical mixture of its eigenstates and the use of averaging via the trace operation rely on Born's rule, postulate 5, which can be derived as follows.

In the classical case, the symmetry among indistinguishable events is due to subjective ignorance. Classically, there is no objective, physical basis for the symmetry and, hence, for objectively equal probabilities. In quantum physics, envariance is an objective symmetry that leads to probabilities of mutually exclusive outcomes such as the orthogonal states deduced earlier from the repeatability postulate.

Suppose that S starts as $|\rightarrow\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$, so interaction with A yields $|\rightarrow\rangle = (|\uparrow\rangle|A_\uparrow\rangle + |\downarrow\rangle|A_\downarrow\rangle)/\sqrt{2}$. For such an 'even' states (= states with equal absolute values of coefficients), all measurement outcomes are equally probable:
The unitary swap $s = |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|$ exchanges the states in S:

$$|\uparrow\rangle|A_\uparrow\rangle + |\downarrow\rangle|A_\downarrow\rangle \xrightarrow{s} |\downarrow\rangle|A_\uparrow\rangle + |\uparrow\rangle|A_\downarrow\rangle. \quad (0.0.7)$$

Probabilities in A are unchanged by this swapping, , so the observation probabilities $p_\uparrow$ and $p_\downarrow$ have been exchanged. Next we swap the records in A (r):

$$|\downarrow\rangle|A_\uparrow\rangle + |\uparrow\rangle|A_\downarrow\rangle \xrightarrow{r} |\downarrow\rangle|A_\downarrow\rangle + |\uparrow\rangle|A_\uparrow\rangle. \quad (0.0.8)$$

That swap restores the original preswap state. This implies $p_\uparrow = p_\downarrow = 1/2$. This can easily be generalized for N symmetric state cases.
[**C**] What has been shown is: if all the coefficients have the same absolute value, then the probabilities of all the basis states must be identical.

As with the uncertainty principle, the indeterminacy of outcomes was a consequence of knowing something else—the whole entangled state. The objective indeterminacy of S or A and the equiprobability of $|\uparrow\rangle$ and $|\downarrow\rangle$ follow.

For an environment comprising many subsystems (formally, an environment expressible as a tensor product of subsystem Hilbert spaces), the initial state $(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|E_0^1, E_0^2, \cdots\rangle$ evolves into

$$|\Upsilon_{SE}\rangle = \alpha|\uparrow\rangle|E_\uparrow^1, E_\uparrow^2, \cdots\rangle + \beta|\downarrow\rangle|E_\downarrow^1, E_\downarrow^2, \cdots\rangle(10) \quad (0.0.9)$$

The state $|\Upsilon_{SE}\rangle$ represents many records inscribed in environmental fragments. We all monitor our world indirectly, eavesdropping on the environmental fragments. As a consequence, the state of S can be found out by many observers—independently and without disturbing S. That redundancy is how evidence of objective existence arises in our quantum world.

An environment fragment F acts as an apparatus with a possibly incomplete record of S. When $E \setminus F$ is traced out, SF decoheres, and the reduced density matrix describing the joint state of S and F is

$$\rho_{SF} = \alpha^2|\uparrow\rangle\langle\uparrow||F_\uparrow\rangle\langle F_\uparrow| + \beta^2|\downarrow\rangle\langle\downarrow||F_\downarrow\rangle\langle F_\downarrow| \quad (0.0.10)$$

in close analogy with equation fr6. When $\langle F_\uparrow|F_\downarrow\rangle = 0$, F contains a perfect record of the preferred states of the system.

The number of copies of the data in F about pointer states is the measure of objectivity; it determines how many times information about S can be extracted from F. The central question of quantum Darwinism is thus: What fraction of F does one need to sample if the goal is to find out about S?

Decoherence can, under the right conditions, lead to "quantum spam" as # E imprints of pointer states are broadcast through the environment. Many observers can independently access those imprints, which ensures the objectivity of pointer states of S.

Repeatability is key.
[**C**] Or the states that allow repeatability is what we objectively observe.
Collectively, the environmental fragments act like the apparatuses. The no-cloning theorem restricts the ability to make copies, but copying is possible when the states to be copied are all orthogonal (see Physics TODAY, February 2009, page 76). There is no literal collapse. However, as a result of decoherence by $E \setminus F$, an observer monitoring the records imprinted on fragments of E will see only one branch, not a superposition of branches.

For an environment comprising many subsystems (formally, an environment expressible as a tensor product of subsystem Hilbert spaces),

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|E0\rangle \to \alpha|\uparrow\rangle|E\uparrow\rangle + \beta|\downarrow\rangle|E\downarrow\rangle \quad (0.0.11)$$

The outcome represents many records inscribed in environmental fragments. As a consequence, the state of S can be found out by many observers—independently and without disturbing S. That redundancy is how evidence of objective existence arises in our quantum world.

An environment fragment F acts as an apparatus with a possibly incomplete record of S. When $S \setminus F$ is traced out, SF decoheres, and the reduced density matrix describing the joint state of S and F

$$\rho_{SF} = |\alpha|^2 |\uparrow\rangle\langle\uparrow||F\uparrow\rangle\langle F\uparrow| + |\beta|^2 |\downarrow\rangle\langle\downarrow||F\downarrow\rangle\langle F\downarrow| \tag{0.0.12}$$

When $\langle F\uparrow|F\downarrow\rangle = 0$, T contains a perfect record of the preferred states of the system.

The number of copies of the data in E about pointer states is the measure of objectivity; it determines how many times information about S can be extracted from E.

The mutual information is the information we can get from F_f, where $f = \#F/\#E$, about S: $I(S : F_f) = H(S) + H(F_f) - H(S, F_f)$.[3] In principle, each individual subsystem might be enough to reveal the state of S. In that case, I($S : F_f$) would jump to $H(s)$ at $f = 1/\#E$. Usually, however, larger fragments of E are needed to find out enough about S. The red curve in figure 0.0.1 shows how, after an initial sharp rise, I($S : F_f$) only gradually approaches the classical plateau at $H * S$). As illustrated in the figure, the initial rise is completed at a fraction f_δ, defined with the help of the information deficit δ observers tolerate: $I(S : F_{f_\delta}) = (1 - \delta)H(S)$.

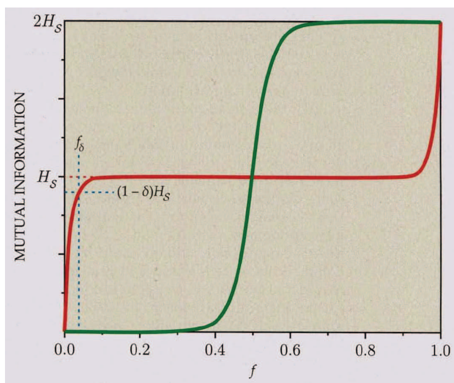

Figure 0.0.1: Information about a system contained in a fraction f of the environment. [Fig. 4 of]

Fig. 0.0.1 Figure 4. Information about a system contained in a fraction f of the environment. The red curve shows a typical result for the mutual information gained via decoherence. Its rapid rise means that a large fraction $(1 - \delta)$ of classically accessible information can be revealed by a small fraction f_δ of the environment. The long classical plateau signifies that additional environmental fragments merely confirm what was already known. The rather different green curve shows the information in the environment for a randomly selected pure state in the system-environment composite.

The inverse of f_δ is the number of records in the environment—the redundancy, R_δ. Many observers can independently access those imprints, which ensures the objectivity of pointer states of S thanks to decoherence. The no-cloning theorem restricts the ability to make copies, but copying is possible when the states to be copied are all orthogonal.

There is no literal collapse. However, as a result of decoherence by $E \setminus F$, an observer monitoring the records imprinted on fragments of F will see only one branch, not a superposition of branches. Such evidence will suggest a quantum jump from a superposition of states to a single outcome

As $f \to 1$, at the last moment I quickly goes up to $2H(S)$, because the SE as a whole is

[3] If S and F are totally uncorrelated $H(S, F) = H(S) + H(F)$. Perfectly correlated, $H(S, F) = H(S) = H(F)$, so classically $0 \le I \le min\{H(S), H(F)\}$. If S and F are pure and entangled, $H(S|F) = 0$, so $H(S, F) = 2H(S)$. Therefore, $I = 2H(S)$. Quantum correlations can be stronger. Decoherence creates a lot of 'entropy.'

pure. However, this indicates why it is so hard to undo decoherence.

If we select a random pure state from SE I behaves as the green curve in Fig. 0.0.1. This is illustrated by

$$(\alpha|\odot\rangle + \beta|\otimes\rangle)|E0\rangle \rightarrow \alpha|\odot\rangle|E\odot\rangle + \beta|\otimes\rangle|E\otimes\rangle. \tag{0.0.13}$$

Not all environments are good witnesses. However, photons excel: They do not interact with air or with each other, and so they faithfully pass on information. A small fraction of a photon environment usually reveals all an observer needs to know. The scattering of sunlight quickly builds up redundancy.[4]

Environments, like air, that decohere S but scramble information because of interactions between subsystems eventually lead to a random state in SE. Quantum Darwinism is possible only when information about S is preserved in fragments of E and so can be recovered by observers.

An observer can get information only about pointer states that remain intact despite monitoring by E: Using the environment as a communication channel comes at the price of censorship. Fractions of E reveal branches one at a time and suggest quantum jumps.

The basic tenets of decoherence have been confirmed by experiment,[5]

[4]C. J. Riedel, W. H. Zurek, Phys. Rev. Lett. 105, 020404 (2010) ; New J. Phys. 13, 073038 (2011); M. Zwolak, C. J. Riedel, W. H. Zurek, Phys. Rev. Lett. 112,140406 (2014); J. K. Korbicz, P. Horodecki, R. Horodecki, Phys. Rev. Lett. 112, 120402 (2014).

[5]M. Brune et al., Phys. Rev. Lett. 77, 4887 (1996); C. J. Myatt et al, Nature 403, 269 (2000); L. Hackermiller et ah, Nature 427, 711 (2004).

Decoherence, einselection, and the quantum origins of the classical

Wojciech Hubert Zurek RMP 75 715 (2003)

717L In essence, the many-worlds interpretation does not address, but only postpones, the key question. The original many-worlds interpretation does not address the preferred-basis question posed by Einstein.

e Witt honestly writes, “But how does the mean square deviation become so small in the first place? Why is a large value of the mean-square deviation of a macroscopic observable virtually never, in fact, encountered in practice? $\cdots$ a proof of this does not yet exist. It remains a program for the future.”

717R Environment can destroy coherence between the states of a quantum system. This is decoherence. According to quantum theory, every superposition of quantum states is a legal quantum state. This egalitarian quantum principle of superposition applies in isolated systems. However, not all quantum superpositions are treated equally by decoherence. There are states that remain stable in the presence of environment.

Einselection is this decoherence-imposed selection of the preferred set of pointer states that remain stable in the presence of the environment. Einselection is an accepted nickname for environment-induced superselection (Zurek, 1982). Decoherence and einselection are two complementary views of the consequences of the same process of environmental monitoring.

The idea that the “openness” of quantum systems might have anything to do with the transition from quantum to classical was ignored for a very long time. Environment distills the classical essence from quantum systems

(1) In quantum physics, “reality” can be attributed to the measured states.
718L (2) Information transfer usually associated with measurements is a common result of almost any interaction of a system with its environment.

It is difficult to catch einselection in action. Einselection is a quantum phenomenon. Its essence cannot even be motivated classically. The quantum nature of decoherence and the absence of classical analogs are a source of misconceptions.
Our aim is to explain why the quantum universe appears classical when it is seen “from within.” Our experience of the classical reality does not apply to the universe as a whole, seen from the outside, but to the systems within it. Schrödinger equation describes the
718R deterministic evolutions

$$|t\rangle = e^{-iHt/\hbar}|0\rangle. \tag{0.0.14}$$

Decoherence makes the many-worlds interpretation complete: It allows one to analyze the universe as it is seen by an observer, who is also subject to decoherence.

719L Envariance offers a new fundamental insight into what is information and what is ignorance in the quantum world.

720L **Quantum Measurements**

The quantum c-NOT is defined by the following control bit kets

$$|0c\rangle|a\rangle \quad \rightarrow \quad |0c\rangle|a\rangle \tag{0.0.15}$$
$$|1c\rangle|a\rangle \quad \rightarrow \quad |1c\rangle|\neg a\rangle \tag{0.0.16}$$

720R Here $|\neg a\rangle$ depends on the choice of the basis.

Let $|\pm\rangle = (1/\sqrt{2})(|0\rangle \pm |1\rangle)$. Then,

$$\begin{aligned} |\pm\rangle|+\rangle &= \frac{1}{2}(|0\rangle \pm |1\rangle)(|0\rangle + |1\rangle) = \frac{1}{2}(00 \pm 10 + 01 \pm 11) \\ \rightarrow & \quad \frac{1}{2}(00 \pm 11 + 01 \pm 10) = \frac{1}{2}(00 \pm 10 + 01 \pm 11) = \frac{1}{2}(|0\rangle \pm |1\rangle)(|0\rangle + |1\rangle) = |\pm\rangle|+\rangle \end{aligned} \tag{0.0.17}$$

$$\begin{aligned} |\pm\rangle|-\rangle &= \frac{1}{2}(|0\rangle \pm |1\rangle)(|0\rangle - |1\rangle) = \frac{1}{2}(00 \pm 10 - 01 \mp 11) \\ \rightarrow & \quad \frac{1}{2}(00 \pm 11 - 01 \mp 10) = \frac{1}{2}(00 \mp 10 - (01 \mp 11)) = \frac{1}{2}(|0\rangle \mp |1\rangle)(|0\rangle - |1\rangle) = |\mp\rangle|-\rangle \end{aligned} \tag{0.0.18}$$

In the complementary basis $\{|+\rangle, |-\rangle\}$, the roles of the control and of the target are reversed. The former target (basis $\{|0\rangle, |1\rangle\}$)—represented by the second ket above— remains unaffected, while the state of the former control (the first ket) is conditionally flipped. The time evolution $\rightarrow$ is governed by

$$H_{int} = g|1\rangle\langle 1|_S|-\rangle\langle -|_A = \frac{g}{2}|1\rangle\langle 1|_S[1 - (|0\rangle\langle 1| + |1\rangle\langle 0|)]_A \tag{0.0.19}$$

because $(0-1)(0-1) = (00 + 11 - 10 - 01)$. $\{|0\rangle, |1\rangle\}_S$ is unaffected by this interaction, since $|0\rangle\langle 0|_S$ and $|1\rangle\langle 1|_S$ are invariant. The c-NOT requires $gt = \pi/2$.

721L The states $\{|+\rangle, |-\rangle\}$ are also intact. Hence, when the apparatus is prepared in a definite phase state (rather than in a definite pointer/logical state), it will pass its phase on to the system. Hence, when the apparatus is prepared in a definite phase state (rather than in a definite pointer/logical state), it will pass its phase on to the system. The question "what measures what?" (decided by the direction of the information flow) depends on the initial states.

721R The measurement process is described by an interaction Hamiltonian between the system and the apparatus. This could be a product of the system observable and some sort of shift operator for the apparatus pointer.

Effect of chaos
The wave packet becomes rapidly delocalized in a chaotic system, and the correspondence
728L between classical and quantum is quickly lost. Flagrantly nonlocal Schrödinger-cat states appear no later than t_h, and this is the overarching interpretational as well as physical problem. In the familiar real world we never seem to encounter such smearing of the wave function even in the examples of chaotic dynamics where it is predicted by quantum theory.

732R Premeasurement:

$$|\Psi_{SA}\rangle|\epsilon_0\rangle = \left(\sum_j \alpha_j |s_j\rangle|A_j\rangle\right)|\epsilon_0\rangle \rightarrow \sum_j \alpha_j|s_j\rangle|A_j\rangle|\epsilon_j\rangle = |\Phi_{SAE}\rangle. \tag{0.0.20}$$

The decohered density matrix describing the SA pair is then diagonal in product states:

$$\rho^D_{SA} = Tr_E|\Phi_{SAE}\rangle\langle\Phi_{SAE}| = \sum_j |\alpha_j|^2|s_j\rangle\langle s_j||A_j\rangle\langle A_j|. \tag{0.0.21}$$

Preservation of the SA correlations is the criterion defining the pointer basis.
By definition, pointer states preserve correlations in spite of decoherence, so that any observable A codiagonal with the AE interaction Hamiltonian will be pointer observable.

733L Einselection as the selective loss of information.
The establishment of a measurementlike correlation between the apparatus and the environment changes the density matrix from the premeasurement ρ^P_{SA} to the decohered ρ^D_{SA}:

$$\rho^P_{SA} = \sum_{ij} \alpha_i \alpha_j^* |s_i\rangle\langle s_j||A_i\rangle\langle A_j| \to \rho^D_{SA} = \sum_j |\alpha_j|^2 |s_j\rangle\langle s_j||A_j\rangle\langle A_j|. \quad (0.0.22)$$

733R Selective loss of information everywhere except in the pointer states is the essence of einselection.

734L

$$I^P(S,A) = H(\rho^P_S) + H(\rho^P_A) - H(\rho^P_{SA}) = -2\sum_j |\alpha_j|^2 \log|\alpha_j|^2, \quad (0.0.23)$$

$$I^D(S,A) = H(\rho^D_S) + H(\rho^D_A) - H(\rho^D_{SA}) = -\sum_j |\alpha_j|^2 \log|\alpha_j|^2. \quad (0.0.24)$$

This follows from the simple tracing, and teh purity of the premeasurement state. Notice that I^P is nonclassical. Decoherence reduces this to the clasically allowed value I^D. Unless this classical upper bound is realized there is no probabilities for the distinct apparatus pointer states.

737L Einselection is responsible for the classical structure of phase space.
The second law of thermodynamics can emerge from the interplay of classical dynamics and quantum decoherence, with entropy production caused by information eeleakingff into the environment.

749R The objective existence of states can be defined operationally by considering two observers. When an observer can consistently determine the state of a system without changing it, that state, by our operational definition, will be said to exist objectively. The environment E acts as a persistent observer, dominating the game with frequent questions, always about the same observables, compelling both observers R (record keeper) and S (spy) to focus on the einselected states. Real observers are forced to perceive the universe the way we do: We are a part of the universe, observing it from within.

750L Hence, for us, environment-induced superselection specifies what exists.
Einselected states are insensitive to measurement of the pointer observables— they have already been measured by the environment.

750R Premeasurement happens at t_0:

$$\left(\sum_i \alpha_i |s_i\rangle\right)|A\rangle \to \sum_i \alpha_i |s_i\rangle|A_i\rangle. \quad (0.0.25)$$

In practice the action is usually large enough to accomplish amplification.

For a real apparatus, interaction with the environment is inevitable. Only the einselected memory states (rather than their superpositions) will be useful for (or, for that matter, accessible to) the observer. The decoherence time scale is very short compared to the time after which memory states are typically consulted (i.e., copied or used in information processing),

751L Decoherence leads to classical correlation,

$$\rho_{pre} = \sum_{i,j} \alpha_i \alpha_j^* |s_i\rangle\langle s_j||A_i\rangle\langle A_j| \rightarrow \sum_i |\alpha_i|^2 |s_i\rangle\langle s_i||A_i\rangle\langle A_i| = \rho_D, \tag{0.0.26}$$

following an entangling premeasurement.

Axiom of quantum measurement

(i) The states of a quantum system S are associated with the vectors $|\psi\rangle$, which are the elements of the Hilbert space H_S that describes S.
(ii) The states evolve according to $i\hbar\partial_t|\psi\rangle = H|\psi\rangle$, where H is Hermitian.
(iii a) Every observable O is associated with a Hermitian operator O.
(iii b) The only possible outcome of a measurement of O is an eigenvalue o_i of O.
(iv) Immediately after a measurement that yields the value o_i the system is in the eigenstate $|o_i\rangle$ of O.
(v) If the system is in a normalized state $|\psi\rangle$, then a measurement of O will yield the value o_i with the probability $p_i = |\langle o_i|\psi\rangle|^2$.

The first two axioms make no reference to measurements.

The two key issues are the projection postulate, implied by a combination of (iv) with (iii b), and the probability interpretation, axiom (v).

We note that the above Copenhagen-like axioms presume the existence of quantum systems and of classical measuring devices. This (unstated) axiom (ϕ) complements axioms (i)-(v).

Our version of axiom (ϕ) posits that the universe consist of quantum systems, and asserts that a composite system can be described by a tensor product of the Hilbert spaces of the
751R constituent systems.[6] Our goal is to describe measurements in a quantum theory without collapse, to use axioms (ϕ), (i), and (ii) to understand the origin of the other axioms.

752L Axiom (iiia): Observables are Hermitian; this is already virtually assumed when we write down the interaction Hamiltonian as on p721R.
Axiom (iiib): Observation causes entanglement (superposition of outcomes): this is the measurement problem.

To address it, we assume that the record states $|A_i\rangle$ are einselected. Hence,
(i) Following the measurement, the joint density matrix of the system and the apparatus decoheres, (0.0.25). consequently, observation does not alter the system density matrix.
(ii) Einselection restricts states that can be read off as if they were classical to pointer states. Thus, Axiom (iii b) is then a consequence of the effective classicality of the pointer states, the only ones that can be found out without being disturbed. They can be consulted repeatedly and remain unaffected under the joint scrutiny of the observers and of the environment (Zurek, 1981, 1993a, 1998a).
[**C**] This means observables are self-adjoint, but the converse is not generally true.

Axiom (iv): immediate repeatability:

752R In the presence of einselection immediate repeatability would apply only to the records made in the einselected states. Other apparatus observables lose correlation with the state of the system on the decoherence time scale. In the effectively classical limit it is natural to demand repeatability extending beyond that very brief moment. This demand makes the role

[6] Axioms (iii)-(v) contain many idealizations.

of einselection in establishing axiom (iv) evident.

753L For probability to be effective for us at the very least the set of events Ω exists independently of the information at hand. Eigenstates of the density matrix do not supply such events. Eigenstates of density matrices in general do not supply the probabilities independent of the eigenstates. Thus, we cannot discuss probabilities of given elementary states.

753R Notice that derivations of Born's rule that employ reduced density matrices are open to the charge of circularity (Zeh, 1997).

"My aim here is to look at the origin of ignorance, information, and, therefore, probabilities from a very quantum and fundamental perspective."

754L Envariance of pure states is a symmetry conspicuously missing from classical physics. Ignorance is a consequence of invariance. Thus, QM with envariance concept can introduce different kind of probability different from the standard argument based on Gleason's theorem.[7]

Envariance: Environment-assisted invariance is a symmetry exhibited by a system S correlated with another system (environment E). Let u_S (resp., u_E be be a transformation acting only on the Hilbert space of the system (resp of the environment) alone. $|se\rangle$ is envariant wrt u_S if there is u_E such that

$$u_E u_S |se\rangle = u_E |X\rangle = |se\rangle. \tag{0.0.27}$$

ψ_{SE} is assumed to be pure (or purified as usual). Use the Schmidt basis:

$$|\psi_{SE}\rangle = \sum_k \alpha_k |s_k\rangle |e_k\rangle \tag{0.0.28}$$

754R Phases associated with the Schmidt basis are obviously envariant. However, we cannot generally undo the effect on E by manipulating S. [**C**] Generally speaking a large system is hard to be envariant.

Theorem 6.1: A local description of the system S entangled with a causally disconnected environment E must not depend on the phases of the coefficients α_k in the Schmidt decomposition of $|\psi_{SE}\rangle$.

[**C**] We could say E introduces random elements that alter phases at least.

It follows that all the measurable properties of S are completely specified by the list of pairs $\{|\alpha_k|; |s_k\rangle\}$. Phases of $|\psi_{SE}\rangle$ can be arbitrarily changed by acting on E alone.

Information contained in the "database" $\{|\alpha_k|; |s_k\rangle\}$ implied by Theorem 6.1 is the same as in the reduced density matrix of the system ρ_S. Although we do not yet know the probabilities of various $|s_k\rangle$, the preferred basis of S has been singled out; Schmidt states (sometimes regarded as instantaneous pointer states) play a special role as the eigenstates of envariant transformations. Moreover, probabilities can depend on $|\alpha_k|$ (but not on the phases). We

[7] **Theorem** [Gleason] Suppose H is a separable Hilbert space of complex dimension at least 3. Then, for any quantum probability measure on the lattice Q of self-adjoint projection operators on H there exists a unique trace class operator W such that $P(E) = Tr(WE)$ for any self-adjoint projection E in Q.
The lattice of projections Q can be interpreted as the set of quantum propositions, each proposition having the form $a \le A \le b$, where A is the measured value of some observable on H (given by a self-adjoint linear operator).

still do not know that $P_k = |\alpha_k|^2$.[8]

755L Lemma 6.2: All of the unitary envariant transformations of ψ_{SE} have Schmidt eigenstates. [Demo] The proof relies on the fact that other unitary transformations would rotate the Schmidt basis, $|s_k\rangle \rightarrow |\tilde{s}_k\rangle$. The rotated basis becomes a new "Schmidt," and this fact cannot be affected by unitary transformations of E, by state rotations in the environment. But a state that has a different Schmidt decomposition from the original $|\psi_{SE}\rangle$ is different. Hence a unitary transformation must be codiagonal with the Schmidt basis of $\psi_{SE}$to leave it envariant.

Envariance under swap:

$$u_S(k \leftrightarrow j) = e^{i\phi_{kj}} |s_k\rangle\langle s_j| + H.c. \tag{0.0.29}$$

If $|\alpha_k| = |\alpha_j|$, then this can be undone by environmental swap

$$u_S(k \leftrightarrow j) = e^{i\phi_{kj} - \phi_k - \phi_j + 2\pi l_{kj}} |e_k\rangle\langle e_j| + H.c. \tag{0.0.30}$$

A swap followed by a counterswap exchanges coefficients of the swapped states in the Schmidt expansion, (0.0.28). Hence, for $|\alpha_k| = |\alpha_j|$ $|\psi_{SE}\rangle$ is envariant under swap.

Observers can know perfectly the quantum joint state of SE, yet be provably ignorant of S. Consider a measurement carried out on the state vector of SE from the point of view
755R of envariance:

$$|A_0\rangle \sum |s_k\rangle|e_k\rangle \rightarrow \sum |A_k\rangle|s_k\rangle|e_k\rangle \sim |\phi_{SAE}\rangle \tag{0.0.31}$$

Suppose the observer knows she is dealing with

$$|\varphi_{SE}\rangle = |s_J\rangle|e_J\rangle. \tag{0.0.32}$$

Then, she knows the system is in the state $|s_J\rangle$, and can predict the outcome of the corresponding measurement on S. $|A_J\rangle$ will be the memory state of hers in the future. This state is not envariant under swap.

By contrast,

$$|\psi_{SE}\rangle = \sum e^{i\phi_k} |s_k\rangle|e_k\rangle \tag{0.0.33}$$

is envariant under swap. This allows the observer (who knows the joint state of SE exactly) to conclude that the probabilities of all the envariantly swappable outcomes must be the same. The observer cannot predict his memory state after the measurement of S because he knows too much: the exact combined state of SE.

For completeness, we note that when there are system states that are absent from the above sum, i.e., states that appear with zero amplitude, they cannot be envariantly swapped with the states present in the sum. Of course, the observer can predict with certainty that he will not detect any of the corresponding zero-amplitude outcomes. For, following the measurement that correlates memory of the observer with the basis $\{|s_k\rangle\}$ of the system, there will be simply no terms describing observer with the record of such nonexistent states of S. This argument about the ignorance of the observer concerning his future state, concerning the outcome of the measurement he is about to perform, is based on his perfect knowledge

[8] How can we connect α_k to probability at all? Even in the Born case, it was forced upon him by empirical results. How can we replace the empirical part with pure logic without extra assumption?

of a joint state of SE. Probabilities refer to the guess the observer makes on the basis of his information before the measurement about the state of his memory—the future outcome—after the measurement. Since the left-hand side of Eq. (0.0.31) is envariant under swaps of
756L the system states, the probabilities of all the states must be equal. Thus, by normalization, $p_k = 1/N$.

Wigner's friend: The postmeasurement state is envariant, if swaps involve jointly the state of the system and the correlated state of the memory:

$$u_{AS}(k \leftrightarrow j) = s^{i\phi_{kj}} |s_k, A_k\rangle\langle s_j, A_j| + Hc. \tag{0.0.34}$$

Thus if another observer ("Wigner's friend") was getting ready to find out, either by direct measurement of S or by communicating with observer A, the outcome of A's measurement, he would be (on the basis of envariance) provably ignorant of the outcome A has detected, but could be certain of the AS correlation.

Note that our reasoning does not really appeal to the information lost in the environment in the sense in which this phrase is often used. Perfect knowledge of the combined state of the system and the environment is the basis of the argument for the ignorance of S alone. For entangled SE, perfect knowledge of SE is incompatible with perfect knowledge of S. This is really a consequence of indeterminacy; joint observables with entangled eigenstates such as ψ_{SE} simply do not commute (as the reader is invited to verify) with the observables of the system alone.

The case of unequal coefficients can be reduced to the case of equal coefficients.

757L Relative frequency from envariance:
757R We emphasize that one could not carry out the basic step of our argument—the proof of the independence of the likelihoods from the phases of the Schmidt expansion coefficients—for an equal-amplitude pure state of a single, isolated system. The problem with

$$|\psi\rangle = N^{-1/2} \sum_k e^{-\phi_k} |k\rangle \tag{0.0.35}$$

is the accessibility of the phases. Consider, for instance, $|\psi\rangle \sim |0\rangle + |1\rangle - |2\rangle$ and $|\psi'\rangle \sim |2\rangle + |1\rangle - |0\rangle$. In the absence of decoherence the swapping of k's is detectable. Interference measurements (i.e., measurements of the observables with phase-dependent eigenstates$|1\rangle + |2\rangle$, $|1\rangle - |2\rangle$, etc.) would have revealed the difference between $|\psi\rangle$ and $|\psi'\rangle$. Loss of phase coherence is essential to allow for the shuffling of the states and coefficients.
[**C**] Thus we need a device to nullify the significance of phases for equal probability. Envariance is needed. However, envariance enforcing environment looks as a deus ex machina.

758L Frequency interpretation of probability has been popular in QM. "However, the infinite size of the ensemble necessary to prove this point is troubling (and unphysical) and taking
758R the limit starting from a finite case is difficult to justify." Also Cox is mentioned: the rule is only classical.

In comparison with all of the above strategies, "probabilities from envariancef" is the most radically quantum, in that it ultimately relies on entanglement.
The insight offered by envariance into the nature of ignorance and information sheds new light on probabilities in physics. The (very quantum) ability to prove the ignorance of a

part of a system by appealing to perfect knowledge of the whole may resolve some of the difficulties of the classical approaches.

Environment as a witness
The emergence of classicality can be viewed either as a consequence of the widespread dissemination of the information about the pointer states through the environment, or as a result of the censorship imposed by decoherence.

759L A complementary approach focuses not on the system, but on the records of its state spread throughout.
one can ask what states of the system are easiest to discover by looking at the environment
The environment is no longer just a source of decoherence, but acquires the role of a communication channel with basis dependent noise that is minimized by the preferred pointer states.

How can objective existence—the e"reality" of the classical states—emerge from purely epistemic wave functions?

States that are recorded most redundantly in the rest of the universe (Zurek, 1983, 1998a, 2000) are also the easiest to discover

Environmental monitoring creates an ensemble of "witness states" in the subsystems of the environment the environment that allows one to invoke some of the methods of the statistical interpretation.

759R Quantum Darwinism

$$|\psi_{SE}\rangle = a|\uparrow\rangle|000...\rangle + b|\downarrow\rangle|111...\rangle = a|\uparrow\rangle|E\uparrow\rangle + b|\downarrow\rangle|E\downarrow\rangle \tag{0.0.36}$$

The basis $u|E\uparrow\rangle$ and $|E\downarrow\rangle$ of S is singled out by the redundancy of the record. In contrast,

$$|\psi_{SE}\rangle = |\odot\rangle(a|000...\rangle + b|111...\rangle)/\sqrt{2} + |\otimes\rangle(a|000...\rangle - b|111...\rangle)/\sqrt{2} = a|\odot\rangle|E\odot\rangle + b|\otimes\rangle|E\otimes\rangle$$

$\langle E\odot|E\otimes\rangle = |a|^2 - |b|^2$ (not even orthogonal) and states$|\odot\rangle, |\otimes\rangle$ cannot be easily inferred from the environment. And even when $\langle E\odot|E\otimes\rangle = 0$ the record in the environment is fragile. Only one relative phase distinguishes $|E\odot\rangle$ and $|E\otimes\rangle$ in contrast with multiple records of the pointer states in $u|E\uparrow\rangle$ and $|E\downarrow\rangle$.

Redundancy of the record in the environment allows for a trial-and-error indirect approach while leaving the system untouched.

The measurement of n environment bits in a Hadamard transform of the basis $\{|0\rangle, |1\rangle\}$, Eq.(0.0.37) yields an algorithmically random sequence. On the other hand (0.0.36) gives a definite 0 or 1.

The state of the form of (0.0.36) can serve as an example of amplification. The generation of redundancy through amplification brings about the objective existence of the otherwise subjective quantum states. States $|\uparrow\rangle$ and $|\downarrow\rangle$ of the system can be determined reliably from a small fraction of the environment. By contrast, to determine whether the system was in a state $|\odot\rangle$ or $|\otimes\rangle$ one would need to detect all of the environment. Objectivity can be defined as the ability of many observers to reach consensus independently. Such consensus concerning states $|\uparrow\rangle$ and $|\downarrow\rangle$ is easily established—many ($\sim N/n$) observers can independently measure fragments of the environment.

Action distance
the total action necessary to undo the distinction between the states of the environment corresponding to different states of the system.
$|\uparrow\rangle$ and $|\downarrow\rangle$: we must flip the spins one by one.
$|\odot\rangle$ or $|\otimes\rangle$: only one flip of the phase is needed.
760L The action distance is a metric on the Hilbert space.

States of the system that are recorded redundantly in the environment must have survived repeated instances of environment monitoring, and are obviously robust and predictable.

760R Consensus and algorithmic simplicity
The complexity of the observation of must be small for good states. States of the system that are recorded redundantly in the environment must have survived repeated instances of environment monitoring, and are obviously robust and predictable.

Sequences of states of environment subsystems correlated with pointer states are mutually predictable and hence, collectively algorithmically simple.
The most direct measure of the reliability of the environment as a witness is the information-theoretic redundancy of einselection itself.

Information-theoretic redundancy is defined as the difference between the least number of bits needed to uniquely specify the message and the actual size of the encoded message.

$$R = I(S, E)/H(S). \tag{0.0.37}$$

761R Its max value is the number of copies of the system in the environment. The symmetric I is not accessible,

$$J(E, S) = H(E) - H(E \,|\, S) \tag{0.0.38}$$

is considered instead (classically, this is I as well).

The observer (or the apparatus, or anything effectively classical) is continuously monitored by the rest of the universe. Its state is repeatedly forced into the einselected states,
762R and very well (very redundantly) known to the rest of the universe.

In essence, macroscopic systems are open, and their evolution is almost never unitary.
Records maintained by the observers are subject to einselection. In a binary alphabet decoherence will allow for only the two logical states and prohibit their superpositions

An observer perceiving the universe from within is in a very different position than an experimental physicist studying a state vector of a quantum system. In a laboratory, the Hilbert space of the investigated system is typically tiny. Such systems can be isolated, so that often the information loss to the environment can be prevented. Then the evolution is unitary. The experimentalist can know everything there is to know about it.

763L The redundancy ratio of the records R is a measure of this objectivity

763R The problem with all of them is that the resulting histories are very subjective: Given an initial density matrix of the universe it is in general quite easy to specify many different, mutually incompatible consistent sets of histories. This subjectivity leads to serious interpretational problems

766R Localization is obviously a challenge to the quantumclassical correspondence, However, the localization time scale is $\sim 1/h$, so quite different $\log h$, and we may ignore this effect for macrosystems.

767L Both groups found that, as a result of spontaneous emission, localization disappears, although the two studies differ in some of the details